A Stochastic User Equilibrium Formulation for the Random Regret Minimization-based Route Choice Model

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ABSTRACT

This paper formulates a static stochastic user equilibrium (SUE) problem in which the Random Regret Minimization (RRM) model is used for route choices. The RRM approach assumes that individuals minimize anticipated regret, rather than maximize expected utility, when choosing among alternative routes.

The cost function for the RRM model is not separable, and so a Variational Inequality approach is adopted to formulate the problem. A path-based algorithm is applied to solve the RRM-SUE problem based on the Method of Successive Averages (MSA). The paper illustrates the algorithmic implementation on a real-world network and discusses the trade-offs and differences between the proposed model and SUE based on random utility models.

Keywords: Random Regret Minimization; Multinomial Logit; Route Choice; Stochastic User Equilibrium; Traffic Assignment.

INTRODUCTION

The recently proposed Random Regret Minimization (RRM) model (*1*) constitutes an alternative to Random Utility Maximization (RUM) models of travel choice. The RRM approach assumes that individuals minimize anticipated regret – rather than maximize expected utility – when choosing routes. Regret is postulated to result from the situation where one or more non-chosen alternatives perform better than a chosen alternative in terms of one or more attributes. The RRM model features MNL-type choice probabilities and can be estimated using conventional discrete choice software packages. In a number of recent empirical studies, the RRM paradigm (and particularly its MNL-model form) has been shown to provide a useful representation of behavior in a number of travel decision making contexts including route, departure time, destination, parking lot, travel information acquisition and vehicle type purchase choices (*2*,*3*).

This paper advances the application of the RRM model in the context of user equilibrium traffic assignment. A Stochastic User Equilibrium (SUE) formulation for the RRM model (its MNL-model form) is presented, implemented and tested. Specifically, this paper formulates and applies a mathematical problem whose solution corresponds to the SUE of an RRM-based route choice model.

Several RUM-based route choice models were developed in the literature to overcome the deficiencies of the basic RUM-MNL model form for route choice modeling, in particular to account for the similarity among overlapping routes (*4*). One group of models is based on the generalized extreme value (GEV) theory (*5*), e.g. the cross-nested logit (CNL) model (*6*) and the paired combinatorial logit (PCL) model (*7*). These models capture the similarity among routes through the structure of the error component of the utility function. Another group of models is obtained by modifying the systematic part of the utility function to account for route overlapping. This approach retains the simple closed-form structure of the MNL model. Models in this group include the C-logit model (*8*) and the path-size logit (PSL) model (*9*). All the models presented above maintain the assumption of RUM-based decision-making. In contrast, this paper derives the SUE formulation in the context of a new, non-utility-based (but regret-based) decision rule.

It should be noted upfront that while the RUM-SUE problem can be formulated as an optimization problem assuming that the cost function is link-separable, the RRM-SUE problem cannot be formulated in the same way, because the cost function in this case is not separable. As will be discussed later in this paper, we solve this issue by adapting a Variational Inequality (VI) formulation to the RRM-SUE context.

The rest of this paper is organized as follows. The next section presents the RRM route choice model (more specifically, its MNL-model form) and provides a brief comparison to RUM's MNL model. The subsequent section formulates the RRM-SUE problem and adapts a path-based algorithm to solve the problem. Results of its application and a comparison with RUM-SUE are illustrated for a simple grid network and for the well-known Winnipeg network. The last section discusses the results and presents directions for further research.

THE RRM ROUTE CHOICE MODEL (MNL-MODEL FORM)

From here on, we will for reasons of brevity and ease of communication use the terms RRM and RUM to refer to their respective MNL-model forms.

For a given network, the cost ('disutility') $c_{k,pq}$ of path k connecting origin p to destination q is generally assumed to be a linear combination of the link costs as follows:

$$c_{k,pq} = \sum_a \delta_{ak,pq} t_a(v_a) \quad (1)$$

Where: v_a is the flow on link a; $t_a(v_a)$ is the travel time on link a, which is assumed to depend only on the flow on link a; $\delta_{ak,pq}$ is the link-path indicator, which equals 1 if link a is a part of path k from p to q, and 0 otherwise. While travel costs may depend on other attributes beyond travel time, they are used interchangeably in this paper.

The well-known RUM route choice model expresses the route flows as follows:

$$h_{k,pq} = g_{pq} P_{k,pq} = g_{pq} \frac{\exp(-\theta c_{k,pq})}{\sum_l \exp(-\theta c_{l,pq})} \quad (2)$$

Where g_{pq} is the total demand for trips between p and q in the period of analysis; $h_{k,pq}$ is the flow on path k from p to q. The positive parameter θ represents a measure of the dispersion among drivers: small values of θ indicate a large perception variance among drivers. As θ increases, the variability among drivers decreases, and the corresponding equilibrium flows approach those of the deterministic user equilibrium.

In an RRM formulation (*1*), the regret c of path k from p to q is computed by comparing the cost of this path to the costs of the other alternative routes as follows:

$$c_{k,pq} = \sum_{l \neq k} \ln\left(1 + \exp\left[\sum_a \delta_{ak,pq} t_a(v_a) - \sum_a \delta_{al,pq} t_a(v_a)\right]\right) \quad (3)$$

Note that this formulation leaves out the random error associated with a path's regret. Various assumptions regarding the random regret term can be made. This paper focuses on the MNL form of the RRM model, in which the random regret term is distributed such that the negative of the error term has an i.i.d. Extreme Value Type I-distribution. It may be noted that random errors in an RRM-model are formulated and interpreted differently than those in an RUM model: in an RRM model they represent unobserved regret (which in turn is a function of cost comparisons), while in a RUM model they directly represent unobserved costs. Stated simply, in the RRM-model the error is not about perception errors at the cost-level, but at the level of cost-comparisons A more in-depth discussion of the rationale behind and the properties of the RRM-model can be found in (*10*), which also provides an overview of the empirical comparisons of the performance of RRM and RUM-based models.

This formulation indicates that the regret of a specific path decreases when it compares favorably to other paths and increases when its travel costs are larger compared to alternative paths. In the case that all path costs are equal their regrets will also be equal and travelers will be indifferent to the choice among them. The corresponding route flows are obtained as in Equation 2, but using the regret costs given by Equation 3 as follows:

$$h_{k,pq} = g_{pq}P_{k,pq} = g_{pq}\frac{\exp\left(-\theta\sum_{l\neq k}\ln\left(1+\exp\left[\sum_a \delta_{ak,pq}t_a(v_a)-\sum_a \delta_{al,pq}t_a(v_a)\right]\right)\right)}{\sum_k \exp\left(-\theta\sum_{l\neq k}\ln\left(1+\exp\left[\sum_a \delta_{ak,pq}t_a(v_a)-\sum_a \delta_{al,pq}t_a(v_a)\right]\right)\right)} \tag{4}$$

In the binary choice case, Equation 4 is identical to the RUM case in Equation 2,. Assuming 2 alternatives, k and l, for each origin-destination pair pq, the route choice probabilities are:

$$P_{k,pq} = \frac{\exp\left(-\theta\ln\left(1+\exp\left[\mathrm{c_{k,pq}}-\mathrm{c_{l,pq}}\right]\right)\right)}{\exp\left(-\theta\ln\left(1+\exp\left[\mathrm{c_{k,pq}}-\mathrm{c_{l,pq}}\right]\right)\right)+\exp\left(-\theta\ln\left(1+\exp\left[\mathrm{c_{l,pq}}-\mathrm{c_{k,pq}}\right]\right)\right)} \tag{5}$$

After some manipulations, the binary logit expression is obtained:

$$P_{k,pq} = \frac{1}{1+\left(\exp\left[\mathrm{c_{l,pq}}-\mathrm{c_{k,pq}}\right]\right)^{-\theta}} \tag{6}$$

The detailed proof can be found in reference [*1*].

The behavioral intuition behind the RRM model in Equation 3 is as follows: it is assumed that the traveler compares a considered path with all other paths in terms of their respective costs. In case the considered path has a lower cost than another path with which it is compared, there is no regret. In case the path with which the considered path is compared has a lower cost than the regret for the considered path equals the difference in costs. Note that this behavioral intuition in fact translates into a regret-function $c_{k,pq} = \sum_{l\neq k}\max\{0,\sum_a \delta_{ak,pq}t_a(v_a) - \sum_a \delta_{al,pq}t_a(v_a)\}$, rather than the function presented in Equation 3. However, due to the presence of the max-operator, this latter function is discontinuous and therefore not differentiable around zero, which poses theoretical and practical problems for the model estimation. The logsum function used in Equation 3 provides a continuous approximation. Further discussion of this logsum form and an illustration of the close approximation it provides of the max-based formulation can be found in (*1*).

We here provide a concise discussion of the properties of the RRM model in the single-attribute case. We consider a choice among three parallel routes, assuming that the costs of all three routes are independent of the flows. The costs on routes A and B are 16 minutes and 18 minutes, respectively. The cost of route C is varied from 15 minutes to 19 minutes. The dispersion parameter $\theta=1$ in all cases. First, choice probabilities are plotted for the three routes in Figure 1 for the RRM and RUM models as a function of the travel cost on route C. The market shares computed by the RUM model are shown as solid lines, and the RRM market shares are shown as dotted lines. The results show that the RRM model predicts that when the travel costs of route C decrease, it attracts more market share (when compared to the RUM model) from the route with higher costs (B) than from the faster route (A). Furthermore, the sensitivity of the RRM choice probabilities to the travel cost is higher than that of the RUM model. When travel times on route C are high RUM predicts a higher share for route C than RRM. This trend is reversed when the route C becomes more attractive. Both these results are consistent with the general properties of the RRM model (*1*), which penalizes poor performance more heavily than RUM and rewards a strong performance more substantially compared to RUM models.

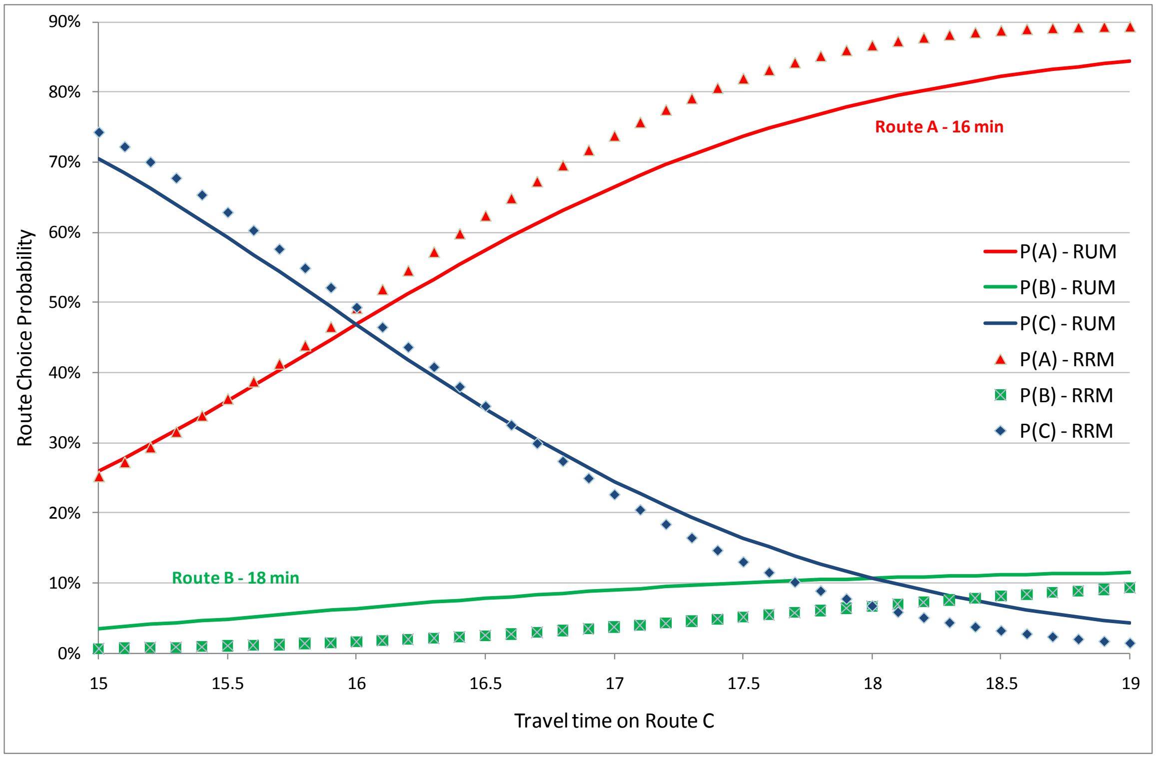

Figure 1: choice probabilities generated by RUM (solid lines) and RRM (dashed lines) for three routes with different travel times

It is important to note that the higher sensitivity of the RRM model cannot be eliminated by tuning the scale of the utilities in the RUM-model. To demonstrate this, Figure 2 shows the difference between the RUM and RRM models as a function of the value of the dispersion parameter θ in the RUM model (θ=1 in the RRM model) for the case that the travel time on route C is 17 minutes. The results suggest that for route C, a RUM model with θ=1.15 would generate the same choice probability as in the RRM model for Route C; an MNL model with θ=1.25 is needed to approximate the RRM choice probability for route A, and θ=1.5 is needed to approximate the RRM choice probability for route B. The fact that these values differ substantially across routes, suggests that the difference between RUM and RRM cannot be eliminated simply by tuning the dispersion parameters.

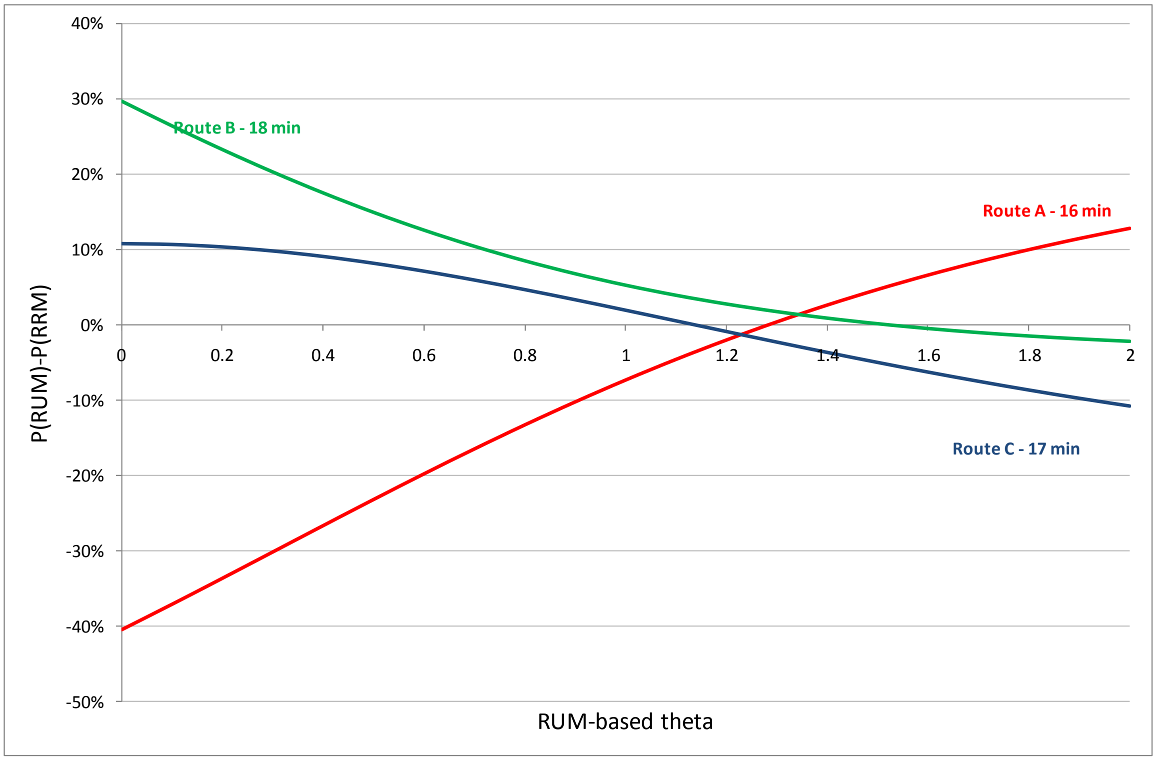

Figure 2: differences between RUM based and RRM based choice probabilities for each route, as a function of the RUM dispersion parameter

Finally, it is useful to observe the change in the differences in the choice probabilities predicted by RRM and RUM as a function of the dispersion parameter, when it is constrained to be equal across the two model types. These differences are shown in Figure 3. This figure shows that when θ is large or close to zero, RRM choice probabilities tend to the RUM choice probabilities. This is in line with expectations: when θ is very large, route choices become deterministic and the fastest route is chosen by all travelers. When θ approaches zero, route choices are fully random for both models and the market shares are equal for all alternatives. However, for intermediate values of θ, there are clearly differences between the two model types. In this particular example, these differences reach a maximum of almost 10% market share approximately when θ=0.75.

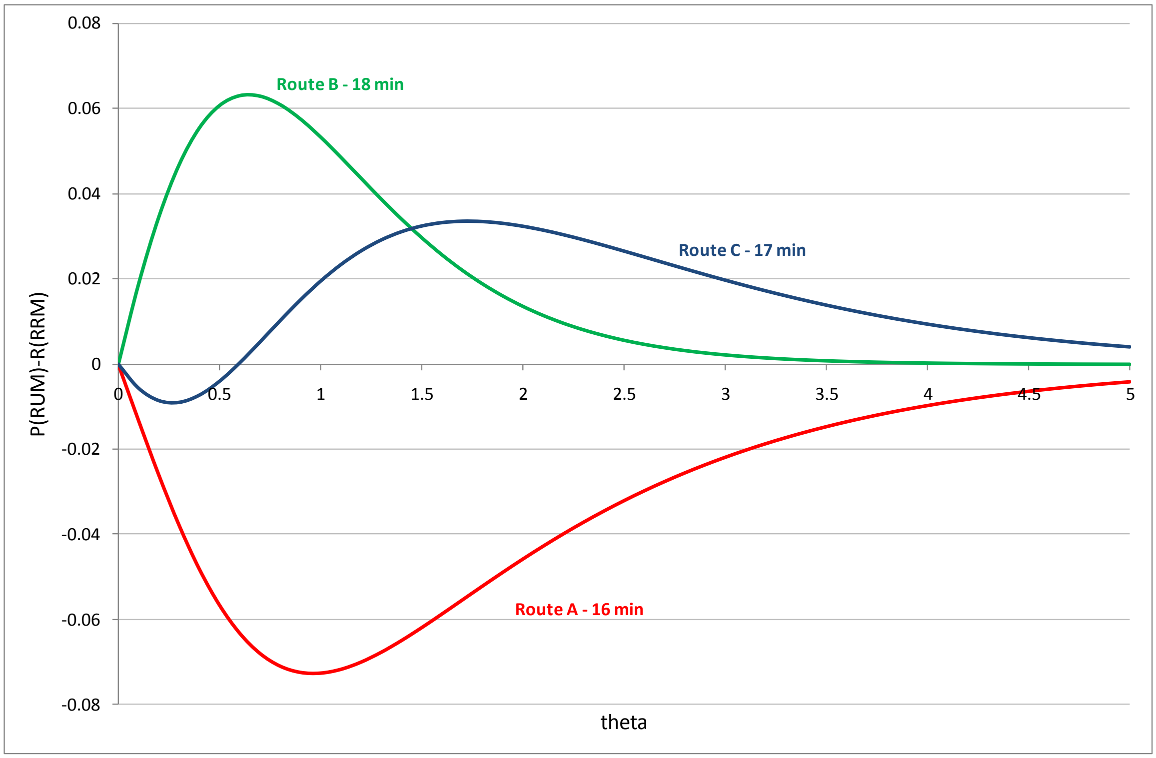

Figure 3: difference between RUM-choice probabilities and RRM-choice probabilities for different values of theta (theta constrained to be equal for RRM and RUM)

In summary, the RRM and RUM models may yield significantly different market shares when the dispersion parameter does not have an extreme value. For a given value of θ, RRM tends to allocate higher market shares to the best routes compared to RUM at the expense of the worst routes.

THE RRM-SUE PROBLEM

Model Formulation

The concept of SUE is defined in (*11*). At SUE, no driver can improve his/her perceived travel time by unilaterally changing routes. The stochastic user equilibrium is mathematically represented as follows:

$$f_{k,pq} = g_{pq} P_{k,pq} \qquad (7)$$

$$P_{k,pq} = P\left(c_{k,pq} + \varepsilon_{k,pq} \leq c_{l,pq} + \varepsilon_{l,pq}, \forall l \in K_{pq}\right) \qquad (8)$$

Where $f_{k,pq}$ is the flow on path *k* connecting origin *p* and destination *q* and K_{pq} is the set of paths connecting origin-destination (O-D) pair *pq*.

The first SUE models used either the simple MNL or the more complex Multinomial Probit (MNP) as route choice models. An optimization formulation for the

MNL-SUE problem is provided in (*12*), in which the solution of the minimization problem is given by the MNL route choice model. Given the non-closed mathematical formulation for the MNP, the Method of Successive Averages (MSA) was proposed to solve the MNP-SUE problem (*13*). Additional equilibrium models based on GEV route choice models were developed in (*14*).

The fact that the cost function defined in Equations 1 and 2 is separable enables the formulation as an optimization program. In contrast, since the RRM cost function expressed in Equation 3 is non-separable (because of the path comparisons), an equivalent optimization program cannot be formulated. Before formulating the RRM-SUE, it is worth pointing out that the definition of the RRM-SUE is slightly different from that of RUM-SUE as RRM-SUE refers to the situation where no driver can decrease his/her perceived regret by unilaterally changing routes.

In order to formulate the RRM-SUE, a Variational Inequality (VI) approach is applied (*15*). The VI is a general problem formulation that encompasses a plethora of mathematical problems, including, among others, nonlinear equations, optimization problems, and fixed point problems (*16*). In geometric terms, the classical VI formulation states that a function F(x*) is "orthogonal" to the feasible set K at the point x*.

$$F(x^*)^T(x - x^*) \geq 0, \forall x \in \mathrm{K} \tag{9}$$

The above formulation is particularly convenient because it allows for a unified treatment of equilibrium problems and optimization problems. In this paper, we use a modified formulation as proposed in (*15*). Let **P** represent the vector of route choice probabilities, where $P_{k,pq}$ is defined as the RRM route choice probability as in Equation 4. The equivalent RRM-SUE model can be formulated as a *VI* problem, which is to find a vector $f^* \in \Omega$, such that:

$$(f - f^*)^T\left(f^* - P\left(c(f^*)\right) \cdot q\right) \geq 0, \forall f \in \Omega \tag{10}$$

Where '•' is the Hadamard product, i.e., $z = x \bullet y \Leftrightarrow z_i = x_i y_i$, $i = 1, 2, \ldots, n$. f^* is a solution of the RRM-SUE model if and only if f^* is a solution of the *VI* problem expressed in Equation 12. The feasible set Ω consists of the following equations:

$$q_{rs} = \sum_h f_{h,rs} \tag{11}$$

$$f_{h,rs} \geq 0 \tag{12}$$

The proof of the above proposition is obtained following (*15*). Firstly, if f^* is a solution of the RRM-SUE model, from the SUE condition in Equation 7, we can see that VI problem above is satisfied naturally. Thus, any equilibrium solution of the RRM-SUE model is a solution of the *VI* problem above. Secondly, suppose f^* is a solution of the *VI* problem; without loss of generality, we fix a path *h* from the set of all routes connecting O-D pair (*r*, *s*) and construct a feasible flow f, such that $f_l^{mn} = f_l^{mn*}$, $(l, m, n) \neq (h, r, s)$ but $f_h^{rs} \neq f_h^{rs*}$. Upon substitution it into Equation 10, one obtains

$(f_h^{rs} - f_h^{rs*})^T (f_h^{rs*} - P_h^{rs}(c^{rs}(f^*)) \cdot q^{rs}) \geq 0$. For every effective route *h* between O-D pair (r,s), there should be $f_h^{rs} > 0$. Therefore, we have $f_h^{rs*} - P_h^{rs}(c^{rs}(f^*)) \cdot q^{rs} = 0$. Thus, the *SUE* condition in Equation 7 is satisfied, and the solution of *VI* problem is the solution of RRM-SUE problem.

If the route travel cost function $c(f)$ is continuous, then the *VI* formulation has at least one solution. According to the assumption of continuity, it is easy to see that $F(f) = f - P(c(f)) \circ q$ is a continuous mapping from Ω to R^n. Since Ω is a nonempty, convex, and compact set, the *VI* problem has at least one solution (*17*).

The *VI* formulation for the RRM-SUE model can be written as a general form:

$$F(f)^T(f - f^*) \geq 0, \forall f \in \Omega \tag{13}$$

where $F(\cdot)$ is a general mapping from Ω to R^n. For the *VI* formulation in Equation 10, the mapping $\left(f - P(c(f)) \circ q\right)$ can be represented by $F(\cdot)$. The *VI* formulation above belongs to a broad category of non-additive traffic equilibrium problems (*18*). The RRM route choice model is non-additive because of the path comparisons in the cost function, as in Equation 3.

It should also be noted that uniqueness of a solution to the VI formulation above depends on the property of mapping $F(\cdot)$. That is, if $F(\cdot)$ is strictly monotone, the VI formulation gives one unique equilibrium solution (*16*). However, the uniqueness of the RRM-SUE model may not be guaranteed due to the non-separable route cost structure.

Path-based Algorithm

An algorithm to solve the RRM-SUE problem is adapted from path-based algorithms discussed in (*19*) and applied to solve the CNL-SUE problem (*20*). Because an optimization problem cannot be formulated, it is not possible to derive an optimal step size or apply Armijo-type rules. As a result, the path-based MSA algorithm is applied to find an approximate solution to the problem.

Routes are generated prior to the assignment, and kept fixed throughout the iterations. After performing an initial loading to obtain a feasible solution, the algorithm successively updates the travel times and path costs, calculates the RRM choice probabilities and assigns the flows on the given routes. The current path-based solution is averaged with the previous iteration, using a predetermined step size (*12*). The path-based MSA algorithm converges to the equilibrium solution, but at the expense of a very large number of iterations (*18*). The equilibrium solution is achieved only if all acyclic paths are included in the path set. Since this is not practical for real networks, a suboptimal solution is achieved. If the path set is fixed, as is the case in this paper, this solution is unique.

Because of the non-optimized step size, the algorithm generally needs a large number of iterations to reach convergence. The stopping criterion for the algorithm is based on the internal inconsistency of the solution:

$$RMSE^{(n)} = \sqrt{\frac{1}{K}\sum_{rs}\sum_{k}\left(h_{k,rs}^{(n)} - f_{k,rs}^{(n)}\right)^2} \quad (14)$$

Where K is the number of routes in the choice sets, n is the iteration counter, $h_{k,rs}$ is path flow computed according to the route choice model for given travel times and $f_{k,rs}$ is the current path flow on the network.

RESULTS

Grid Network

Figure 4 presents a simple grid network. The free-flow travel times and link capacities are respectively indicated in the network. In this example, there are two OD pairs with positive demand: between 1 and 6 (10 units of flow) and between 1 and 9 (20 units of flow). For this simple network, the universal choice set can be generated. It is composed of 3 routes for OD pair 1-6 and 6 routes for OD pair 1-9. A path-based MSA algorithm was used for all models, and the stopping criterion was set to 0.001 maximum RMSE difference between link flows. The following link performance function was used in all the tests:

$$t_a = t_{0a} * \left(1 + 0.6\left(\frac{x_a}{s_a}\right)^4\right) \quad (15)$$

Where t_a is the travel time on link a; t_{0a} is the free-flow travel time on link a; x_a is the flow on link a; s_a is the capacity on link a. It is assumed that path travel times are obtained by summing the travel times of each link that forms the path. The flow x_a is obtained after assigning the path flows for each OD pair with positive flow on the network.

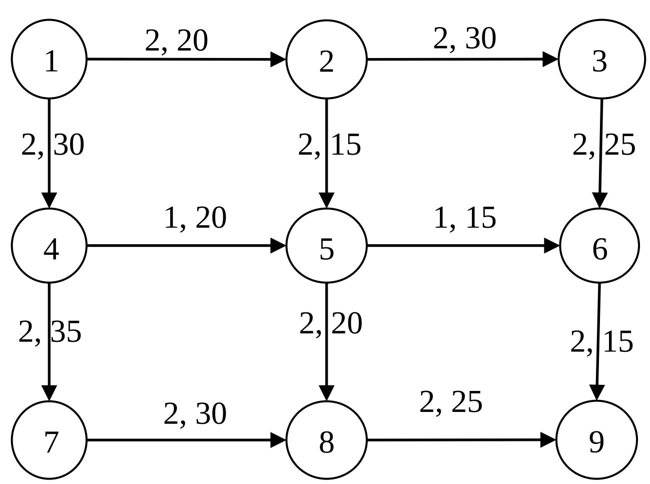

Figure 4. Grid Network

This simple example is provided to illustrate the differences between the RRM and RUM equilibrium results. Note that the uncongested fastest route between 1 and 9 is Route 1-4-5-6-9 and the uncongested fastest route between 1 and 6 is Route 1-4-5-6. Figure 5 compares the path flow probabilities of choosing Route 1-4-5-6-9 according to RUM-SUE and RRM-SUE network equilibrium results, as a function of the dispersion parameter θ. The solid lines represent RUM-SUE results for different demand levels, and the dashed lines represent RRM-SUE results for different demand levels. The total demand for each OD pair is scaled by constant factors (0.6, 0.8, 1.0, 1.2 and 1.4), and for each demand level the equilibrium is computed.

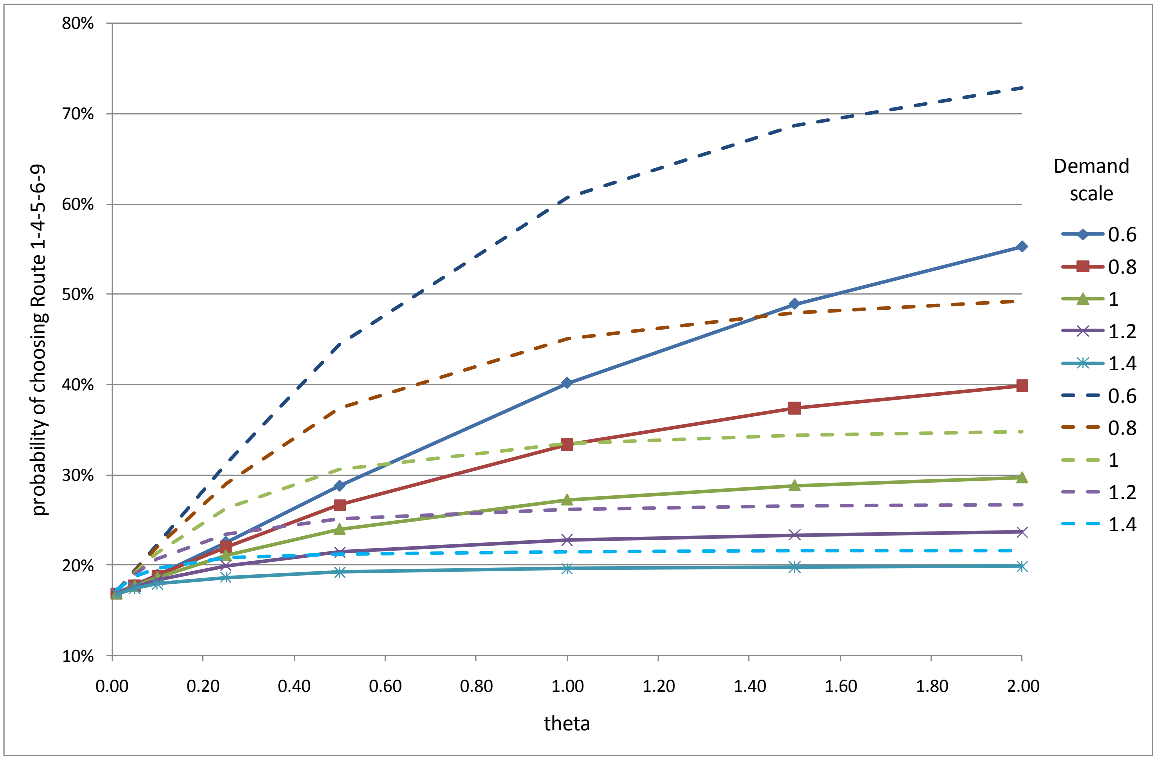

Figure 5. Choice probabilities for Route 1-4-5-6-9 for RUM-SUE (solid lines) and RRM-SUE (dashed lines) and for different demand scale levels.

For a given demand, the probability of choosing Route 1-4-5-6-9 increases with θ. This result is consistent with the theory, since high values of θ indicate low variance with respect to travel time perception. For a given θ, the probability of choosing Route 1-4-5-6-9 decreases for increasing demand. This result is also expected, because as the network becomes more congested, the path travel times tend to be close to each other, lowering the relative attractiveness of Route 1-4-5-6-9. Note, however, that the proportion of flow in this route is always higher than 1/6. This extreme case only occurs when θ is zero, meaning that the travel time variance tends to infinity, and therefore the probability of choosing any one of the 6 routes between origin 1 and destination 9 is equal. In line with expectations and the numerical examples presented above, RRM-SUE results in higher shares for route 1-4-5-6-9 than RUM-SUE for all demand levels. 1-4-5-6-9 is the fastest route for this OD pair, and RRM rewards this path more strongly than RUM. Note however that the difference between RRM flow and RUM flow decreases for increasing demand. This result is specific for the grid network presented above, since the travel times on the alternative routes become close to the fastest route for increasing demand levels. Similar results are obtained for Route 1-4-5-6.

Winnipeg Network

The Winnipeg network database provided in the EMME/2 software (*21*) is used to compare RUM-SUE and RRM-SUE results for a more realistic network. The network is composed of 948 nodes (154 of which are centroids), 2,535 links and 4,345 OD pairs with positive demand. The total demand on the network is 54,459 trips for the AM peak. The volume-delay function for each link is based on the BPR formula with link-specific parameters, calculated from the original EMME/2 data.

Routes were generated prior to the assignment, using a combination of the link elimination method (*22*) and the penalty method (*23*), with a penalty of 5% increase travel time on the shortest path links. Only acyclic paths were considered in these methods. A total of 174,491 unique routes were generated for all OD pairs (average of 40.1 routes per OD pair). The maximum possible number of routes generated for each OD pair was 50. Inspection on the routes generated for the different OD pairs reveals that the choice set used for the analysis includes both completely disjointed routes and very similar routes. This was expected due to the methods (link penalty and link elimination) selected to generate the routes: the link elimination method produces disjoint routes (because of the removal of all links belonging to the shortest path) and the link penalty method produces similar routes because of the low penalty (5% increase link travel time) used to find the subsequent routes. The same choice set was used in previous papers (*24*).

Table 1 shows the effect of the values of the parameter θ on the RMSE of the difference between RUM-SUE and RRM-SUE. The deviation between the two models in Table 1 is measured as follows:

$$RMSE = \sqrt{\frac{1}{K}\sum_{rs}\sum_{k}\left(f_{k,rs}^{(RUM)} - f_{k,rs}^{(RRM)}\right)^2} \qquad (16)$$

Where K is the number of routes in the choice sets. The same formula is used to calculate the deviation at the link flow level. The table includes also results computed for a more restrictive case allowing a mximum of 5 routes per OD pair. Note that in this case 21,723 routes are generated, out of 21,725 possible (4,435 * 5).

Table 1 RMSE between RUM-SUE and RRM-SUE path-flows and link-flows for different values of theta - Winnipeg network

Variable	K	Theta				
		0.01	0.05	0.1	0.5	0.1

Path-flows (Max. 50 routes generated for each OD pair)	174,491	0.297	0.528	0.642	0.936	0.976
Path-flows (Max. 5 routes generated for each OD pair)	21,723	0.280	0.567	0.740	1.054	1.156
Link-flows (Max. 50 routes generated for each OD pair)	2,535	136.765	143.777	126.271	71.756	52.742
Link-flows (Max. 5 routes generated for each OD pair)	2,535	24.481	30.918	30.435	22.787	18.289

The results presented in Table 1 indicate that the path-flow deviation increases with θ. Note that the average demand on a route is about 0.31 for up to 50 routes per OD pair (54,459/174,491), and about 2.51 for up to 5 routes per OD pair. This means that the RMSE for the 5 route case is relatively small, compared to the 50 route case. Nevertheless, in both cases the deviation is high, meaning that RUM-SUE and RRM-SUE produce significantly different path-flows.

In contrast to the path flows, the RMSE link flows do not exhibit a monotonic pattern. This result is difficult to interpret, because many routes have several links in common. Note that RMSE values are higher for the 50 route case, in comparison to the 5 route case. Following the path-flow results, in both choice sets the differences between the two models are significant.

Figure 6 illustrates the link flow difference between RRM-SUE and RUM-SUE results, setting θ equal to 0.5. The green color indicates that RUM link flows are higher than RRM link flows. Note the concentration of RUM link flows around the city center. This is explained by the relatively high number of low-capacity links in the city center, and consequently more congested routes passing through the center. Similar to the three-route example and the grid network example, the RRM route choice model penalizes more heavily the more congested routes, in comparison to the RUM route choice. Therefore, the overall link flow pattern results in more RUM flows in the city center. This interpretation is similar to previous papers that compared RUM-based CNL-SUE and MNL-SUE link flows (*25*).

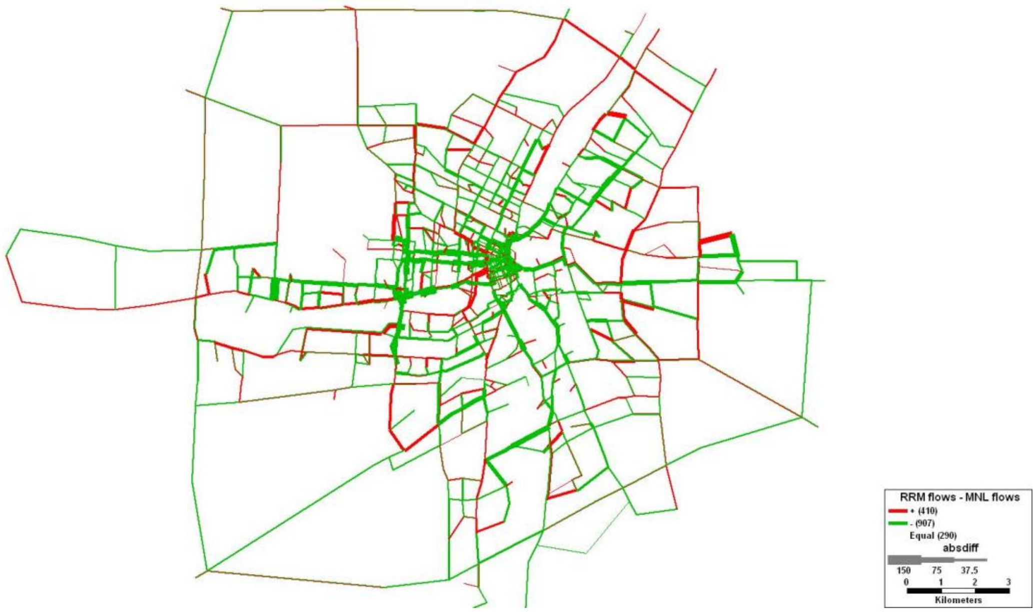

Figure 6 Link Flow Difference between RRM-SUE and RUM-SUE models (θ=0.5).

The computation times for a single iteration of RRM-SUE and RUM-SUE are quite similar, because the additional effort related to path comparisons in RRM-SUE is not time-consuming. In a desktop PC computer - Intel Core 2 Duo CPU (3.0 GHz speed and 4.0 GB RAM) it took 2.2 seconds per iteration. However, RRM-SUE takes more iterations to converge, compared to RUM-SUE. Table 2 shows the number of iterations needed to reach convergence for two criteria (RMSE equal to 0.1 and 0.01, respectively) and for different values of theta, using the Winnipeg network with 5 routes per OD pair.

Table 2. Number of Iterations needed to converge – Winnipeg network

Convergence criterion (RMSE)	Model	Theta				
		0.01	0.05	0.1	0.5	1
0.1	RRM	5	19	31	132	242
0.1	RUM	3	10	17	62	127
0.01	RRM	39	178	301	1261	2348
0.01	RUM	14	90	155	595	1209

SUMMARY AND FURTHER RESEARCH DIRECTIONS

This paper discussed the RRM approach to route choice modeling, and presented a VI formulation for the RRM-SUE model. The results show that the model can be implemented on real-size networks in practice.

The comparison between RUM-SUE and RRM-SUE results, performed for a simple network and for a real-size network, indicates that differences among the equilibrium route flows can be significant and are in line with the differences in behavioral premises underlying the two model paradigms. Depending on the network topology and the number of routes generated, the results may be quite different even at the link flow level.

This study compared the results between RRM and RUM models of the MNL form. Further research is needed to compare RRM and RUM in the context of other route choice model forms, such as C-Logit, PSL or CNL models and to examine their effects on the equilibrium solutions. In addition, it would be interesting to explore how similarity and route-overlap can be modeled in an RRM framework. There are also some other important issues that may affect the performance of the solution algorithm and equilibrium flow patterns, such as various demand levels and different route set generation methods (a priori or column generation). The impacts induced by these issues are worth further investigation. The convergence properties, such as robustness and efficiency, of path-based algorithms for solving equilibrium problems can also be compared in future research.

In addition to these theoretical advances, an obvious direction for further research would be to provide additional empirical testing between RRM-based and RUM-based route-choice models. Preferably, data should be collected and analyzed at the level of the individual traveler's choices (using Stated Preference-surveys or Revealed Preference-datasets) as well as at the level of aggregate network flows.

Furthermore, note that the route choice model considered in this article is a function of travel times only. The formulation of the problem can accommodate additional explanatory variables, similar to the 'generalized cost' variable in deterministic traffic assignment problems. More complex utility functions are yet to be implemented in traffic assignment models. Note however, that the RRM model can easily be formulated at the multi-attribute level – in fact, its original formulation as presented in (1) is multi-attribute. The assumption in a multi-attribute setting is that attribute-level regrets are summed over all attributes, so that associated parameters reflect the relative importance of corresponding attributes. Note that in this regard the multi-attribute RRM-model resembles the multi-attribute RUM-model (more specifically: its linear-additive model form), which also assumes that attribute-level utilities are summed over attributes to arrive at alternative-level utilities.

Finally, it may be noted that the results presented in this article are based on several assumptions common to simple equilibrium models: static assignment, fixed demand, separable volume-delay function and single-user class. Additional research is needed to extend and verify the RRM-SUE model for more general problems.

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