PROPERTIES OF DYNAMIC FREEWAY NETWORK FLEXIBILITY MODEL

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ABSTRACT

The objective of this study is to investigate the properties of the flexibility model in a network of freeways which conveys high volumes of traffic. For that a Dynamic System Flexibility model was defined and its three major components are evaluated and discussed. The Dynamic System Flexibility measures and estimates the options of drivers traveling between origin–destination pairs (O-Ds) in a freeway network. The three models' components depend on the number of possible and feasible routes between a given O-D pairs; on the common lengths among the possible routes and, on the amount of variability between each route's length and the length of the shortest route. The lengths of the link in the model are perceived lengths and depend on the occurrence of congestion in them.

It was found that the flexibility measure provides a good estimate of the number of options available to drivers in a given network. Moreover, this measure is sensitive to the amount of congestion in the freeway system, and therefore, the model that is proposed is flow-dependent. It was also found that while the common lengths between the routes increase, the flexibility decreases. A pseudo paradox, similar to Braess' paradox, is shown to exist under certain network conditions.

1. INTRODUCTION

Many previous studies examined the flexibility of freeway networks mainly with respect to what happens to the network flow when a link is completely unusable due to major catastrophic events or abnormal behavior during a disaster (*Fang and Wakabayashi, (1)*; *Miller-Hooks et al., (2)*). Other models of network flexibility capture the importance of each link to the overall performance of the network. Nonetheless, since the disruption of a link does not necessarily cause its capacity to drop to zero and the flexibility is dynamic because the state of congestion in each link changes continuously over time, a Dynamic Flexibility Model was proposed by *Sofer et al. (3)*. This model not only captures the topology of the freeway network, but also the existence of congestion that might occur in the links comprising the network, thus taking into consideration the amount of traffic.

Previous studies have not viewed the flexibility of a freeway network as a dynamic issue. Therefore, the Dynamic Flexibility Model enables the evaluation of the system flexibility (under congestion conditions) and to compare several freeway networks for their congestion and flexibility, thus enabling the development of measures to increase flexibility and reduce congestion. The objective of this paper is to investigate the dynamic flexibility model properties and present selected applications, variables and paradoxes that may occur when comparing the flexibility of different networks.

2. PREVIOUS STUDIES

The flexibility of transportation networks has been investigated by number of researchers. Yet, the approach to and definition of flexibility is not uniform among the studies.

Miller-Hooks et al. (2) calculated the resilience of a transportation network in the case of reduced capacity on certain links by calculating the network resilience as the expected value of the ratio between the post-disaster maximum demand that can be satisfied for the origin–destination (O–D) and the original pre-disaster O–D demand for all pairs.

Feitelson and Salomon (4) defined flexibility of transportation networks as the ease with which a network can adjust to changing circumstances and demands. These circumstances can involve infrastructure changes, e.g., the ease and cost of inserting an additional link between nodes, as well as operational changes, e.g., transportation requires coordination among users so a link's use by one user does not prevent its use by others.

Morlok and Chang (5) defined flexibility of transportation systems as the ability of a transport system to accommodate variations or changes in traffic demand. The changes in the network must ensure that a satisfactory level of performance is maintained. In order to define the problem they used linear programing to maximize the network's total cargo traffic, subject to routing and resource constraints. *Sun et al. (6)* used the same approach of defining network flexibility as *Morlok and Chang's (5)* and extended their model to capture uncertainty of traffic patterns or volume–delay functions by adding a stochastic traffic-assignment procedure to eliminate the need for path enumeration and to make the model more useful for large networks.

A different way of defining network flexibility was offered by *Scott et al. (7)* and later by *Sullivan et al. (8)*. They presented an approach for identifying critical links and evaluating network performance, which they called the Network Robustness Index (NRI). This index calculated the "importance" of each link. *Scott et al. (7)* calculated the additional travel time in the equilibrium assignment without one link minus that with this link, hence capturing the importance of this particular link in the overall network. *Sullivan et al. (8)* proposed the Network Trip Robustness (NTR) measure, which allows different networks to be compared by calculating the sum of all NRIs for all links in the network divided by the total demand for all O–Ds in the network. Consequently, networks with different numbers of links, volumes and link capacity can be compared.

Previous studies investigated the importance of each link separately, as Scott et al. (7) did, or calculated the resilience of a network in the case of reduced capacity on certain links, as Miller-Hooks et al. (2) did. No study up to now has calculated the flexibility of a network based on dynamic conditions and a congestion state that may be present at specific times on certain links. Following the definition of NTR proposed by Sullivan et al. (8), and taking into account that demand can vary throughout the day, the flexibility model proposed in this study is a dynamic model that depends on the prevailing flow conditions.

Sofer et al. (3) defined flexibility as a measure that estimates the options drivers have when traveling between O-Ds in a freeway network. These options may change due to existence of congestion that may be present at specific times on certain links, and therefore, the network flexibility is dynamic. The ideal, most flexible network is one that has an infinite number of routes between each O-D, all routes are not common with other routes and are equal in length without congestion. Links that are used by more than one route are termed "common", and the utility of using them is reduced. This phenomenon was investigated by *Cascetta et al. (9)* who presented a method for reducing the utility of overlapping routes. Several route choice models were developed in the literature that account for flow-independent route overlapping. The flow-dependent commonality factor was investigated by *Zhou et al. (10)*, who investigated the stochastic user equilibrium problem with the route choice model based on the C-logit function.

In order to capture the congestion state that may occur in different links, *Sofer et al. (3)* presented the term "perceived lengths". The length of each link is equal to its absolute length only if there is no congestion in the link, since a small decrease in speed relative to the Free Flow Speed (FFS) does not cause congestion and drivers are not likely to be sensitive to this small decrease in speed. Otherwise, when the link is in a congested state, the travel time that drivers experience is longer than it would be in a non-congested situation, and therefore, not only is the speed reduced significantly, but the flow is also unstable. The perceived length of Link j is given by Equation (1).

$$k_j = \begin{cases} \dfrac{l_j}{\alpha + \beta\left(\dfrac{v}{c}\right)^{\theta}}, & \textit{if link } j \textit{ is congested} \\ l_j, & \textit{if link } j \textit{ is not congested} \end{cases} \tag{1}$$

where:

l_j = absolute length of Link j;

k_j = relative length of Link j (i.e., the perceived length);

α, β, θ = parameters estimated from the observations.

The system flexibility model of a given O-D in the study by *Sofer et al. (3)* included three variables: A variable describing the number of routes between a given

O-D, a variable describing the common lengths among all routes, based on the commonality factor, and a variable describing the amount of variability among available freeway routes relative to the shortest route between each O-D. The flexibility model is presented in Equation (2).

$$F_{(O-D)} = \left(1 - \frac{1}{e^{(N-1)/\delta}}\right) \cdot \left(e^{-\mu * \frac{L_{m,C}}{\sqrt[N]{K_{1,NC} * ... * K_{N,NC}}}} \right) \cdot \left(\frac{1}{1 + \mathrm{var}(k_{C_i} / k_{C[\min]})} \right) \qquad (2)$$

where:

- $F_{(O-D)}$ = the flexibility model for a given O-D pair;
- N = number of effective routes between a given O-D (the routes, which are input and known a priori, are generated by the route-choice model);
- $L_{m,C}$ = total relative length along all routes in a given O-D pair that is common to more than one route (i.e., when a link is used by more than one route under normal traffic conditions; if only one route uses a certain link, this link is not included in L_m); the lengths are perceived and calculated according to Equation (1), so that the more severe the congestion in a link, the longer is its relative length.
- $k_{i,NC}$ = absolute length of Route i, which is calculated as the sum of the absolute lengths of all l_j used on Route i;
- k_{c_i} = relative length of Route i between a given O-D, when the lengths of all links belonging to this route are perceived.
- $k_{C[\min]}$ = relative (perceived) length of the shortest route between a given O-D.
- δ, μ = positive parameters to be calibrated. The parameters need to be positive so that the function of the variable describing the number of routes will be monotonically increasing function and the function of the variable based on the commonality factor will be monotonically decreasing function.

Note that in the above formulation, the route set is assumed to be given. For real applications, there is a need to form the route set by a route choice set generation procedure. Once the choice set is formed, it is possible to calculate the network flexibility.

In order to calculate the flexibility at a given moment of all O-Ds in the network, an average of all individual flexibilities is calculated, since the presence of congestion and its severity is taken into account by Equations (1) and (2). The flexibility model for the entire freeway network--i.e., all possible O-D pairs (P)— is the sum of all individual flexibilities for each O-D pair divided by the sum of all possible O-Ds in the system as presented in Equation (2).

$$F_{system} = \frac{\sum_{O-D=1}^{P} F_{(O-D)}}{P} \qquad (3)$$

It should be noted that the system flexibility model, F_{system}, is dimensionless, and therefore one can use it to compare and evaluate different high-speed, high-capacity networks (freeways and toll roads) regardless of their relative size and length.

After sensitivity analysis of several generic examples and estimation of the rate of decline at this stage of the research, it was decided to adopt the value of $\delta = 3$ and $\mu = 0.3$, so that each variable will contribute proportionally and in an unbiased way to the model. It was found that when $\mu > 0.7$, the contribution of the commonality variable had an disproportional impact on this component.

This flexibility model takes into account the prevailing volume and congestion at any given time and, thus, is dynamic in time. The perceived lengths of a system's links and the amount of commonality among alternative routes are key components of the flexibility model.

This paper further examines the flexibility model presented by *Sofer et al. (3)* in order to examine in depth the implementations, definitions, properties and paradoxes that might arise in terms of the flexibility model and its variables and parameters.

3. EXAMINATION OF PERCEIVED LENGTH CHARACTERISTICS

In Equation (1), above, the perceived length is either equal to the absolute length when no congestion occurs, or equal to a longer perceived length beyond the onset of congestion. The threshold used in this research in order to define the onset and duration of congestion is based on the Critical Occupancy Point (COP) according to *Sofer et al. (11)*. This measure combines two kinds of parameters, speed and occupancy, and may be developed from data obtained independently from each freeway sensor because the information on each sensor provides measurements that reflect the flow conditions at that particular location. COP is the inflection point in the relationship of speed as a function of occupancy; i.e., the point at which the function changes from a concave to a convex function; this phenomenon is described in Equation (3).

$$f = S_{\min} + \frac{S_{FFS} - S_{\min}}{1 + e^{\beta(OCC - COP)}} \tag{3}$$

Where:

f = curve-fitted speed;

S_{FFS} = free-flow speed (FFS), calculated from the average low-volume speeds detected by the sensor;

$S_{\min}$ = minimum speed calculated by the sensor for the given observations;

β = parameter describing the steepness of the function;

OCC = occupancy recorded by the sensor;

COP = critical occupancy point.

Prior to the COP, the state of flow is un-congested, and then even a small decrease in speed relative to the FFS does not cause congestion. Drivers are not likely to be sensitive to this small increase in speed. However, beyond the congested flow state, not only is the speed reduced significantly, but the flow is also unstable (stop and go, with substantial fluctuations among small speeds). The system flexibility model proposes that only in a congested state it is needed to increase the lengths of the links.

Pollatschek and Polus (12) investigated the stochastic nature of freeway flow and capacity definitions. They proposed dividing the flow into free-flow, dense-flow and unstable-flow, and found a parabolic relationship between speed vs. volume. The COP occurs before the actual breakdown flow when examining speed vs. volume—specifically, congestion starts at the dense flow region. Therefore, the perceived length of congested links depends on whether the flow in this link belongs to the dense flow or to the breakdown flow. A fitted curve of the perceived length was found according to an estimation based on the best-fit technique applied to observations of speed –volume taken at 5-minute periods from 17 data sets and presented in Equation (4). The parameters for the perceived length in the dense flow are based on the fitted curve of observations in the free flow and dense flow while the parameters for the perceived length in the breakdown flow are based on the fitted curve of observations in the breakdown flow.

$$k_j = \begin{cases} \dfrac{l_j}{\alpha_1 - \beta_1\left(\dfrac{v}{c}\right)^{\theta_1}} \quad , & \text{if link } j \text{ is congested and in dense flow} \\ \dfrac{l_j}{\alpha_2 + \beta_2\left(\dfrac{v}{c}\right)^{\theta_2}} \quad , & \text{if link } j \text{ is congested and in breakdown flow} \\ l_j \quad , & \text{if link } j \text{ is not congested} \end{cases} \tag{4}$$

The parameters α_1, $\beta_{1,}$ θ_1, α_2, $\beta_{2,}$ and θ_2 in Equation (4) are 0.99, 0.32, 2.04, 0.06, 0.39 and 2.03, respectively. A schematic description of the perceived length of a link of one unit length in the different regions is shown in Figure 1.

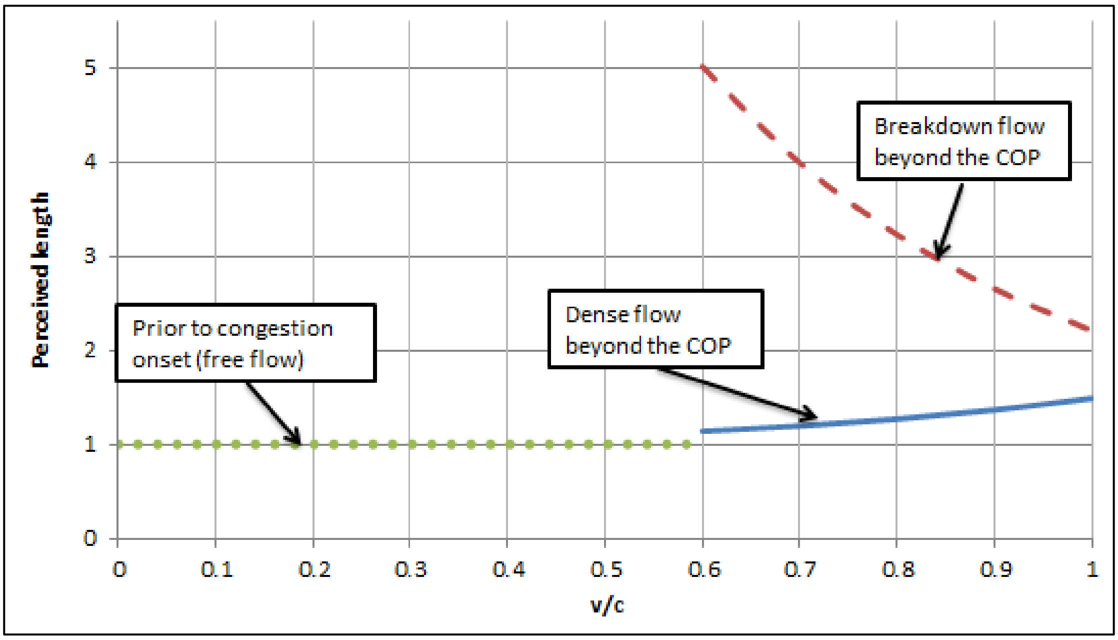

FIGURE 1 – Schematic Description of the Perceived Length of a Link of One Unit Length in the Different Region Flows.

Note that the perceived length increases up to about 1.5 times the actual length in the dense flow zone while it increase between 2.2 to about 5 of the actual length in the breakdown zone.

4. APPLICATION OF THE DYNAMIC SYSTEM FLEXIBILITY MODEL

The flexibility model was implemented on a transportation network consisting of eight major freeways in Israel's central region. The network included 57 nodes, where each node represents a change in traffic conditions. There were 193 O-D pairs resulting from 14 options for entering or exiting the network. A schematic map of the network is presented in Figure 2.

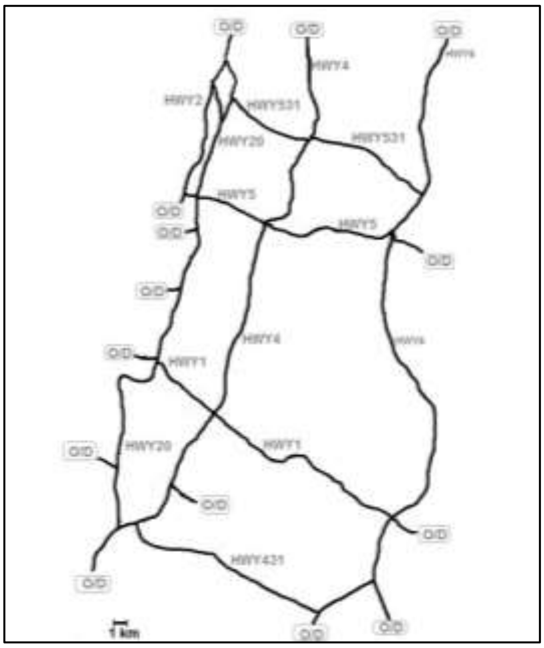

FIGURE 2 – Schematic Map of 8 Major Freeways in Israel's Central Region.

The routes in the freeway system in Figure 2 were determined according to the link elimination procedure, as it appears in the study by *Bekhor et al. (13)*. Static user equilibrium assignment of the demand during rush hour (7 until 9) was found according to the demand of trips at that time and the number of vehicles in each link was known accordingly. In addition, the percent of daily trips during the day in ranges of two hours was known from a national data survey and using that ratio an estimation regarding the number of vehicles within each time zone of two hours during the day was done. Since we only know the length of each link, its capacity and the volume of vehicles in it, and do not know if the flow is in dense zone or breakdown zone, when calculating the perceived lengths we assumed that when v/c is over 0.6, the link is congested, and during rush hour (7-9 am and 3-5 pm), the congested links have breakdown flow while at all other times, congested links are in dense flow. Figure 3 represents the number of congested links in each range of two hours (for example, during 7-9 am there were found 59 congested links in the system) and the flexibility at those ranges as well. The dashed line in Figure 3 is the change in flexibility during the day on the basis of the change in percent of daily trips. The more vehicles in the network, the less flexible is the network and thus drivers face less possible options when passing through the network. Note that the flexibility estimates the entire freeway system and not the number of congested links (flexibility of 32.75% with no links congested vs. flexibility of 32.5% during 13-15pm with 30 links congested in the dense flow).

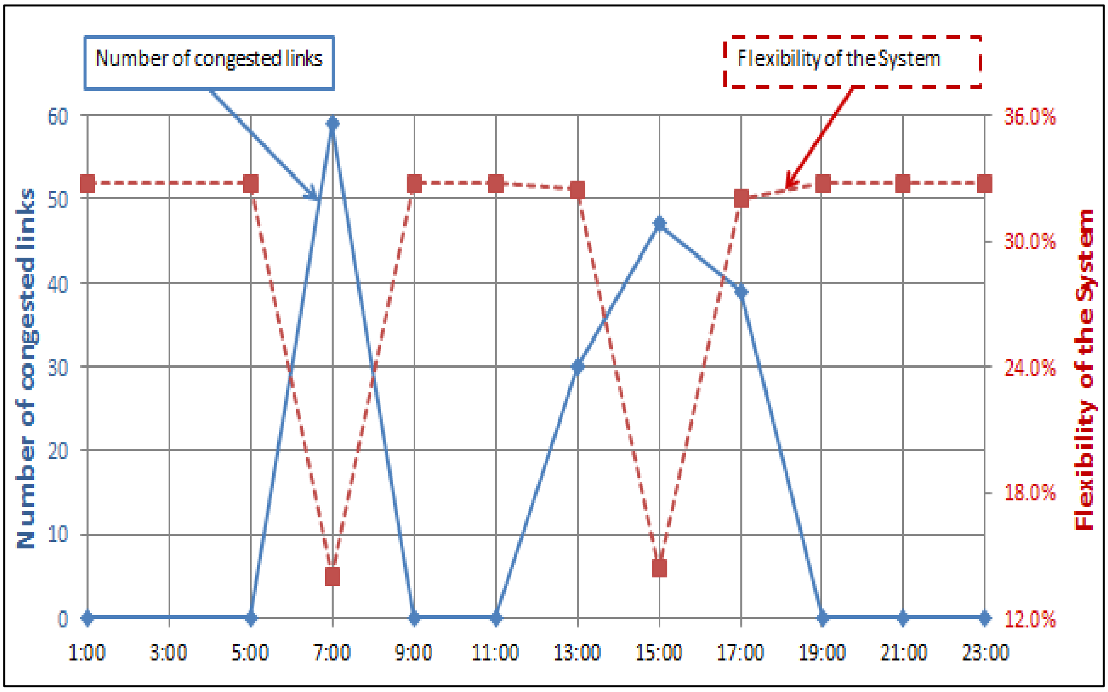

FIGURE 3 – Flexibility of the Network Presented in Figure 2.

When examining the flexibility of different O-Ds separately, the pattern of flexibility that appeared in Figure 3 is not necessarily maintained. The overall flexibility of the system in Figure 2 can decline at times with congestion, while the flexibility of a given Oi-Dj might improve during the same congested time. A closer examination of this outcome shows that this can happen when the links that were congested were the individual links in the given Oi-Dj, and therefore, the variables regarding the number of links and commonality have not changed. Simultaneously, the variability variable improved and the perceived routes were more similar in length, relative to the shortest route in Oi-Dj. This means that the options drivers have during congested times when leaving Oi are more similar, and therefore, they had more alternatives when travelling through the network to Dj at that time, relative to the available options when there was no congestion.

5. FURTHER ANALYSIS OF THE COMMONALITY VARIABLE IN THE FLEXIBILITY MODEL

As mentioned before, the routes in the network presented in Section 4 were found using the link elimination procedure algorithm. The routes between each O-D in the example presented in Figure 2 varied between 4 at the minimum and 15 at the maximum, when the average options that drivers faced in the given network were 7.8 options. There was a clear tendency of the function describing the commonality variable in Equation (2) as function of N to be a decreasing function. This outcome is presented in Figure 4 when examining the flexibility of the network topologically, with all lengths equal to their absolute length and no congestion. The emergence of the decreasing function is logical given that the more routes there are between each O-D, the more likely that there will be common links between them, and therefore, the total length of all perceived common links will increase while the commonality variable will decrease in this situation. In the presence of congestion in certain links, the common variable for the number of routes might increase only its range of values since the sum of all the common links in the numerator may only increase in the situation when no congestion is present in all links, whereas the denominator stays constant regardless of the presence of congestion.

Without contradicting the analysis above, it should be noted that the commonality variable is highly dependent on the way the routes are being built. *Prato and Bekhor (14)* discuss the effect of size and composition of the route choice set, but assuming flow-independent conditions. In this study there is only one algorithm from which the routes were built. The values of the commonality variable might change completely if the routes were built under other constraints, such as that common links are to be avoided as much as possible.

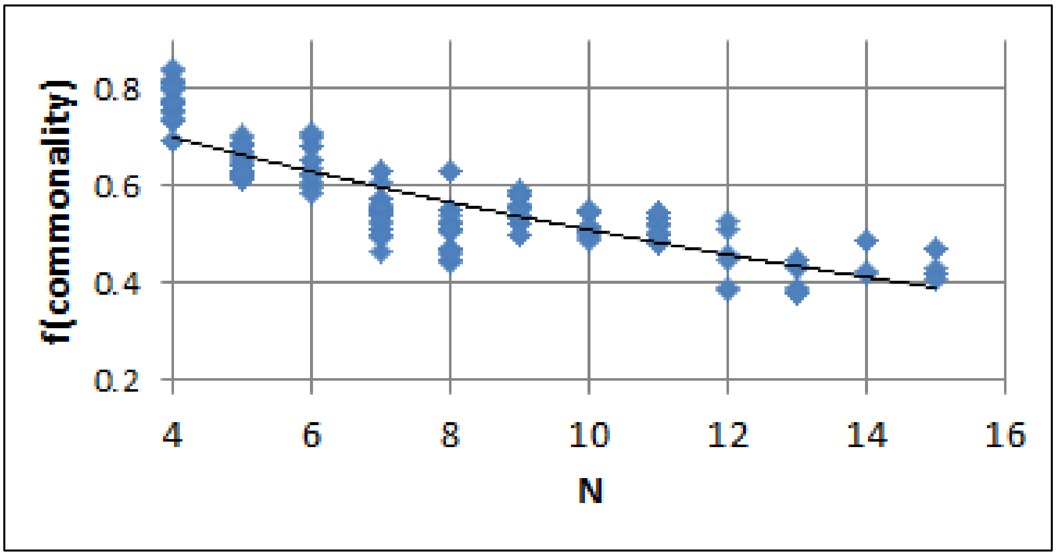

FIGURE 4 – Commonality Variable With No Congestion In All Links In the Routes of Figure 2.

6. THE 'PARADOXES' IN THE SYSTEM FLEXIBILITY MODEL

A paradox might occur when adding routes to a network results in a less flexible network rather than a more flexible network. An example of this is shown in Figure 5, where the numbers on the links represent the links' lengths. The left network represents two routes between an O-D that are equal in length and each route is independent. Therefore, the flexibility of the left network according to Equation (2) when the parameters used are $\delta = 3$ and $\mu = 0.3$ is equal to 28.3%. When adding a link to the network as in the network on the right, three routes emerge and with the addition of the new route, the two original routes now have some common lengths. If both links of 50 unit lengths are connected entirely (and then x=50, in Figure 5), the flexibility declines to 14.3%. The less common lengths there are and the smaller is x in Figure 5, the more flexible is the network on the right. The break-even point—when both networks in Figure 5 have the same flexibility—happens when x=21 and y=21.023, according to Pythagoras' law or $y/_{50} = 0.42$. When any x is smaller than 21 length units, the flexibility of the right network is higher than the left one.

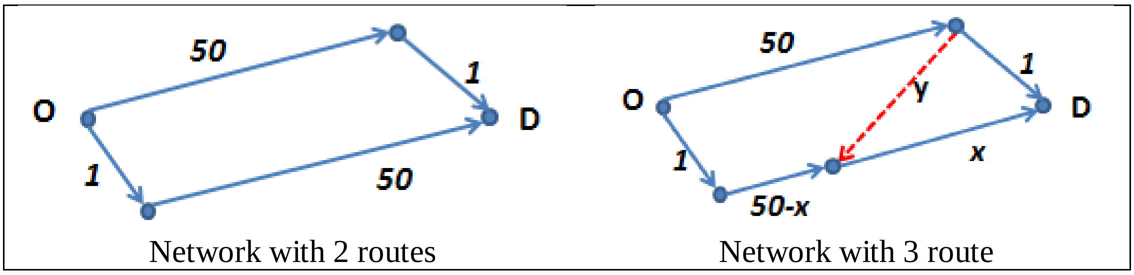

FIGURE 5 – An Example of a Network With the Existence of a Paradox.

Nevertheless, since one link in the length of 50 units will be always common to more than one route even if x=0, the flexibility of a hypothetical network with three equal-length routes, each of which is independent, will always be greater than the network on the right in Figure 5. This break-even point is represented in Figure 6.

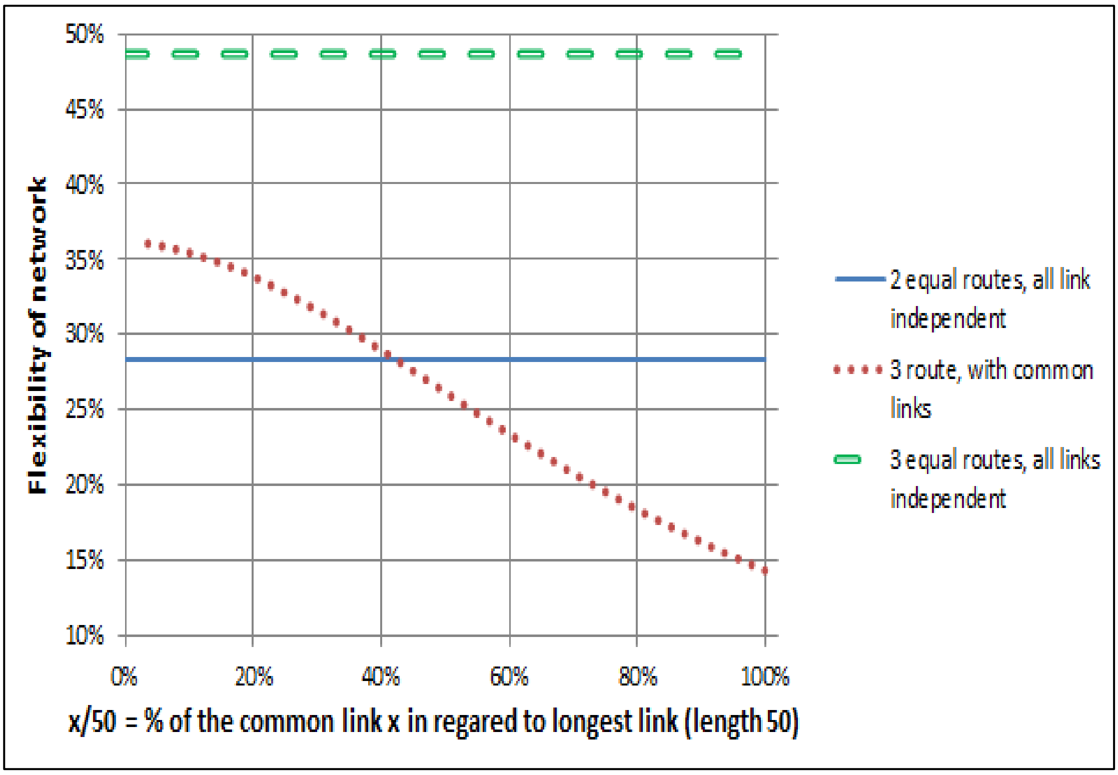

FIGURE 6 – The Impact of x on the System Flexibility (Example Network in Figure 5).

In Figure 5 the ratio of the shortest link in the left hand network to the longest link is $^1/_{50}$. Had this ratio of the shortest link to the longest link been $^1/_{1.51}$ and a diagonal link were added, the network with three routes would be more flexible than the one with two routes and the paradox would not exist when the parameters used in Equation (2) are $\delta = 3$ and $\mu = 0.3$.

When the common lengths in Figure 5 are equal to 100, and thus x=50, any combination of $\delta, \mu > 0$ will result in the paradox where the network with two independent routes is more flexible than the network with three routes having common lengths. However, there are networks in which a paradox will result in some combinations of $\delta, \mu > 0$, and in contrast, no paradox will result with other combinations. An example of this can be seen in the networks presented in Figure 7 where the numbers on the links represent the links' lengths. The grid network on the left has 10 routes to connect the O-D with common lengths equal 42 and all routes are equal in length. Had we added another independent route to the left hand network in Figure 7, of equal length to the 10 given routes, the flexibility would have increased from 47.2% to 47.9%. The network on the right hand side in Figure 7 has an additional link that is common to the two additional routes added to the ten existing ones. Consequently, the common lengths in the right network are equal to 54.16, while the variability variable is smaller than 1 since not all links are equal in length and there exists variability between the 12 routes of the network.

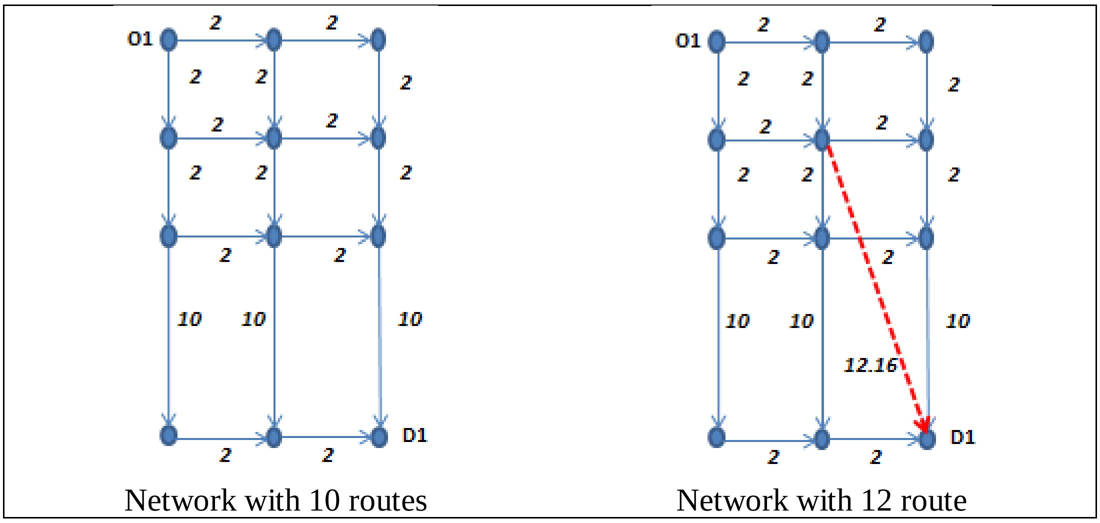

FIGURE 7 – An Example of Networks That May or May Not Fulfill the Paradox.

Figure 8 represents the distribution of $\delta, \mu > 0$ spaces within which the paradox of Figure 7 happens (and then the flexibility of the right network is smaller than the left network in Figure 7) and when it does not exist (and then the flexibility of the left network is smaller than the left network in Figure 7).

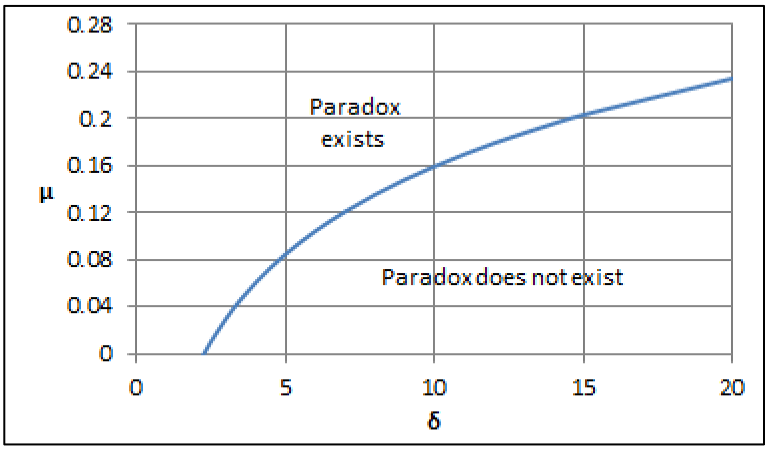

FIGURE 8 – The Impact of the Parameters on Adding an Additional Diagonal Link in a Grid Network (Example in Figure 7).

7. SUMMARY

This paper further examines Dynamic System Flexibility in regard to the options that drivers have when traveling between O-Ds in a freeway network. The options depend on the number of available routes, the common links between more than one route and the variability among available freeway routes relative to the shortest route between each O-D.

It was found that the more routes there are between an O-D, the more the commonality variable will decrease. In addition, there are networks in which adding routes can increase or decrease flexibility, depending on the models' parameters, while in other network flexibility will increase or decrease no matter what parameters are used. The conclusion of this study is that when there is an option of adding a new link, route or network, a close investigation whether or not this link/route increases the system's entire flexibility is essential and the Dynamic System Flexibility Model can assist in such decisions.

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