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Specification and Estimation of Mode Choice Model Capturing Similarity between Mixed Auto and Transit Alternatives

Shlomo Bekhor[1,*] Yoram Shiftan[2,†]

[1] Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel
[2] Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

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Abstract

To analyse the diversion from auto modes to combined modes such as park and ride, it is common to develop mode choice models based on discrete choice theory. In most cases, park and ride is modelled as an access mode to a main transit mode. This paper proposes an approach to test similarities among modes and the appropriate model structure, providing the flexibility for various model structures. The paper explores the capability of recently developed models by specifying their structure to capture the similarities of the combined modes. The paper presents an example with real data to illustrate the methodology application. Estimation results for different model structures including the Multinomial Logit, Nested Logit, Cross-Nested Logit and the Logit Kernel with all of these previous models as kernel are presented. As expected the best estimation results are obtained for the most flexible model, the Logit Kernel with Cross Nested as Kernel.

Keywords: Mode Choice, Multinomial Logit, Nested Logit, Cross-Nested Logit, Logit Kernel, Park and Ride

* Corresponding author, T: +972-48292460, F: + 972-48295708, sbekhor@technion.ac.il
† T: +972-48292381, F: + 972-48295708, shiftan@technion.ac.il

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1 Introduction

In line with sustainable transportation policies there is a special interest in transit development and improvement to attract car users. To analyze the diversion from auto modes to park and ride and kiss and ride modes it is common to develop mode choice models based on discrete choice theory. The definition and structure of the available alternatives for each individual is an important step in the choice modeling process. In cases that the alternatives are unambiguously defined, such as the choice between choosing to travel by car or taking a bus, the definition of the alternative is fairly easy. A more complicated task rises in the case of park and ride and kiss and ride that involve a combination of car and transit modes that is not always well defined. The purpose of this paper is to investigate the similarity of park and ride modes to both auto modes and transit modes in an effort to understand the potential of such modes to attract auto users using different model structures to represent this similarity.

With the continuous development of mass rapid transit systems to provide accessibility to large city centers and reducing auto trips, there is also continuous research on various topics related to park and ride as the mode that is most likely to be used by those who shift from auto to transit. Some research focus on various design issues such as the location and pricing of park and ride facilities (Wang et. al. 2004, Faghri et al. 2002) and various applications around the world (Cairns 1998; Lam et al. 2001; and Seik 1997). However, one of the main issues is forecasting the demand for park and ride (Hole 2004) and understanding the various factors that can affect people to use it (Bos et al. 2004a,b), its potential to reduce road traffic (Parkhurst 2000) and the broader environmental, social and economic effects (Parkhurst and Richardson 2002) as well as detailed estimation of the different decisions involving park and ride. For example, in the PRISM Model developed for the UK Department of Transport, a nested structure representing the access mode to public transport at the higher level and a station alternative choice at the lower level was estimated (Rohr 2007). In this model all public transport modes are represented separately, but can be integrated to reflect mixed-mode journeys.

The most common model structures used in mode choice models are the multinomial logit (MNL) and nested logit (NL) models. The MNL model assumes that the alternatives are independent from each other, and it is generally not used when combined modes are present. The NL model is generally used in cases involving combined modes, in which the alternatives are grouped in nests. However, park and ride can belong to both transit nests sharing unobserved transit attributes and to auto nest sharing unobserved auto attributes. Traditionally, it has been assumed that such combined modes are mainly transit modes sharing more unobserved attributes with other transit modes than with auto modes. Nowadays, however, there are more “premium” transit modes based on light or heavy rail. Thus, the modeling of the choice between such combined modes and pure auto mode should take into account more unobserved attributes than do a "simple" park and ride mode with a transit with walk access mode. This makes the choice of the nest structure to be more ambiguous. There is evidence from the literature that the nest definition is a restrictive assumption in such cases. For example, Forinash and Koppelman (1992) showed that there are multiple nest forms to model the choice of long distance travel between airplane, train, bus and car. More recent examples can be found in Vovsha (1997) and Cascetta and Papola (2003).

In recent years, more general models were developed to better capture the similarity among modes, while keeping the convenient analytical properties of the logit family. Many models were developed applying the Generalized Extreme Value theory of McFadden (1978), such as the Cross-Nested Logit (Vosha 1997), and the Paired Combinatorial Logit (Chu 1989). Other models were developed using a mixture between probit and logit models, such as the Logit Kernel (or Mixed Logit) model providing a more general form that can be used to approximate any of the above model forms (see for example McFadden and Train (2000) and Ben-Akiva and Bolduc (1996)). These more general models have the advantage of being flexible enough to accommodate many forms of similarity among alternatives. Walker et al. (2004) presented several examples to illustrate the potential of the factor analytic Logit Kernel (LK) model. An example of such application can be found in Bekhor et al. (2002), who adapted the LK model for route choice accounting for similarities among different routes sharing common links.

This paper explores the capability of recently developed models by specifying the model structure to capture the similarities of the park and ride mode to either auto or transit modes. The level of similarity and a more accurate representation of the park and ride mode are important to better understand the potential of park and ride to attract auto users. For this purpose we discuss two different model structures, the Cross-Nested Logit as a representative of the closed-form family of models, and the Logit Kernel as representative of more flexible model structures. For completeness we first briefly present these models and then we discuss the specifications end estimation issues of these models to test the similarities of the park and ride and kiss and rides modes to auto and transit modes and their advantages to capture such similarities. The model estimation is performed using stated preference (SP) data collected for the Tel Aviv mass transit system and already used to estimate NL models (Polydoropoulou and Ben-Akiva 2001) for various modes including park and ride and kiss and ride.

2 Selected Choice Models

2.1 The Cross-Nested Logit Model

The Cross-Nested Logit (CNL) model presented in this paper is a generalization of the two-level Nested Logit model. The CNL model structure allows an alternative to belong to more than one nest. Vovsha (1997) presented the derivation of the linear homogeneous CNL probability function. Wen and Koppelman (2001) further developed the Generalized Nested Logit (GNL) model, a generalization of the CNL model. Ben-Akiva and Bierlaire (1999), Papola (2004) presented similar developments of the CNL model. Recently Daly and Bierlaire (2006) developed the Network GEV model, which allows for more flexible structures. All developments were made using McFadden’s (1978) GEV theorem.

The generator function for the GNL model is defined as follows:

$$G(y_1, y_2, \cdots, y_n) = \sum_m \left(\sum_k (\alpha_{mk} y_k)^{1/\mu_m} \right)^{\mu_m} \tag{1}$$

Where: *m* are nests, *k* are alternatives, μ_m is the degree of nesting (specific for each nest), $0 \le \mu_m \le 1$. α_{mk} are the inclusion coefficients allocating alternatives to nests, $0 \le \alpha_{mk} \le 1$.

The inclusion coefficients are subject to a regularity constraint:

$$\sum_m \alpha_{mk} = 1, \quad \forall k \tag{2}$$

The generator function defined in equation (1) satisfies the conditions required for the GEV theorem. As a result, the expression for choice probability can be obtained as follows:

$$P(k) = \sum_m P(m) P(k|m) \tag{3}$$

Where the conditional probability of an alternative *k* being chosen in nest *m* is:

$$P(k|m) = \frac{(\alpha_{mk} \exp(V_k))^{\frac{1}{\mu_m}}}{\sum_l (\alpha_{ml} \exp(V_l))^{\frac{1}{\mu_m}}} \tag{4}$$

Where *l* indicates an alternative. The marginal probability of a nest *m* being chosen is:

$$P(m) = \frac{\left(\sum_k (\alpha_{mk} \exp(V_k))^{\frac{1}{\mu_m}} \right)^{\mu_m}}{\sum_b \left(\sum_k (\alpha_{bk} \exp(V_k))^{\frac{1}{\mu_m}} \right)^{\mu_m}} \tag{5}$$

Where *b* indicates a nest. The probability of choosing alternative *k* depends on three factors: the deterministic component of the utility function V_k, the inclusion coefficients α_{mk} associated with nest *m* that forms the alternative *k*, and the nesting coefficients μ_m. When μ_m is equal to one for all *m* nests, the CNL model collapses to the MNL model.

The formulation of the CNL presented above permits an alternative (in our case, an access mode) to belong to more than one nest (in our case, a main mode). The crossing effect is represented by the inclusion coefficients α_{mk}, $0 \le \alpha_{mk} \le 1$. The Nested Logit model is a special case of the CNL model, in which the coefficients α_{mk} are either zero or one. By assigning only binary values for α_{mk}, an alternative can only belong to one nest, as in the NL model.

2.2 The Factor Analytic Logit Kernel Model

In the Logit Kernel (LK) model, the random utility term is made up of two components: a Probit-like component with a multivariate distribution, and an independent and identically distributed (i.i.d.) Gumbel random variate. The Probit-like term captures the interdependencies among the alternatives. We specify these interdependencies using a factor analytic structure, which is a flexible specification that accommodates different error structures, as was shown by Walker (2001). It also has the ability of capturing complex covariance structures with relatively few parameters.

The LK suffers from the same computational difficulties as pure Multinomial Probit. Programs to estimate these types of models are widely available (see Train et al. (1999)). Bhat (2001) presented the use of intelligent drawing mechanisms known as Halton sequences (in many cases non-random draws or better referred to as "quasi-random" draws). These draws are designed to cover the integration space in a more uniform way, and therefore can significantly reduce the number of draws required. Latest development in these field suggest an array of other "quasi-random" draws including the (t,m,s)-nets (Sandor and Train 2004) and the Modified Latin Hypecube Sampling (MLHS), (Hess et al. 2006)

The general form of the factor analytic Logit Kernel model (in vector notation) following Walker (2001) is:

$$U = \beta\mathbf{X} + \varepsilon = \beta\mathbf{X} + \mathbf{F}\xi + \nu \tag{6}$$

$$\xi = \mathbf{T}\zeta \tag{7}$$

$$\mathbf{cov}(\varepsilon) = \mathbf{F}\mathbf{T}\mathbf{T}^{T}\mathbf{F}^{T} + (g/\mu^{2})\mathbf{I} \tag{8}$$

Where:

β is a vector of unknown parameters (dimension $K\times 1$)
X is a matrix of explanatory variables (dimension $J\times K$)
F is a factor loadings matrix (dimension $J\times M$)
T is lower triangular matrix of unknown parameters (dimension $M\times M$)
ζ is a vector of unknown factors (dimension $M\times 1$)
ν is a vector of i.i.d. Gumbel variables with scale parameter μ (dimension $J\times 1$)
g is the variance of a standard Gumbel variable ($\pi^2/6$)

The elements of F and T may be estimated or specified from data. As presented later in the paper, we will define the F matrix in a convenient way to adapt it to represent the similarities among the various modes in our mode choice model.

To obtain the probability function, we make use of the convenient logit formulation as follows. If the factors ζ are known, the following expression is obtained:

$$\Lambda(i|\zeta) = \frac{\exp(\mu(X_i\beta + F_iT\zeta))}{\sum_j \exp(\mu(X_j\beta + F_jT\zeta))} \tag{9}$$

Where $\Lambda(i|\zeta)$ is the probability to choose alternative *i* given ζ (the vector of unknown factors). The above function is equivalent to the multinomial logit formulation. Since the factors are unknown, the unconditional probability is given by:

$$P(i) = \int_{\xi} \Lambda(i|\zeta) \prod \phi(\zeta) d\zeta \qquad (10)$$

The advantage of the Logit Kernel is that we can estimate the probability function by simulation:

$$P(i) = \frac{1}{R} \sum_{r=1}^{R} \Lambda(i|\zeta^r) \qquad (11)$$

Where *R* is the number of simulation draws.

Note that equation (9) can be replaced by more complex GEV models, such as Nested Logit or Cross-Nested Logit. Estimation results for these models are presented later in the paper.

3. An Illustrative Example

The adaptation of the CNL and LK model structures is illustrated using a real case study. The study is based on the Tel Aviv mass transit proposed system and on stated preference (SP) data that were collected to develop the mode choice model for that project. The data and models developed for this project are described by Polydoropoulou and Ben-Akiva (2001) and the reader is referred to their paper for more details. In our case study there are nine alternative modes as follows: Bus with Walk access (BW), Bus with Kiss and Ride access (BKR), Bus with Park and Ride access (BPR), Rail with Walk access (RW), Rail with Kiss and Ride access (RKR), Rail with Park and Ride access (RPR), Rail with Bus access (RB), Car driver (CD), and Car Passenger (CP). Other modes including taxi, walk and bike were not included in this experiment as they are not common alternative for the suggested mass transit system.

3.1 Data and Survey

The SP questionnaire had five parts. The first part briefly explained the survey and included questions on household composition, respondent characteristics, and a simple trip diary used to select a recent trip for the remainder of the questionnaire. The second part of the questionnaire included questions on the attributes of the selected trip. These responses were used in the SP experimental design to generate the levels of the attributes for the choice experiments. Based on the origin-destination pair of the selected trip, the SP questionnaire program randomly selected one mass transit system alternative. Then, a presentation on the selected mass transit (MT) system alternative was shown. The aim of this presentation was to explain in detail the technologies involved in the MT alternative, to present pictures of the vehicles and stations, and to answer respondents' questions about the MT service. Thus, it was ensured that the

Each SP choice experiment consisted of a sequence of choices including main mode and access mode. The main modes include the alternatives of bus, mass transit, car driver and car passenger. The access modes include the alternatives of walk, park and ride, and kiss and ride for bus access, with the addition of bus for mass transit access. Six SP experiments were administered to each respondent, varying the attribute levels of the access and main mode alternatives. While in each experiment there were a limited number of alternatives to choose from, the survey was designed such that overall all the alternatives had sufficient observations to estimate the various model structures presented in this paper. Transit modes and Kiss and Ride were always available, while car driver and park and ride were available only for individuals that have a driver's license and a car available. For a detailed description of the attributes of the main mode and access mode alternatives see Polydoropoulou and Ben-Akiva (2001) as well as an example of the structure of a main mode choice experiment, as it was presented on the computer screen.

The survey was conducted by personal interviews with laptop computers. The population was sampled using telephone lists as sampling frame. The response rate reported from the survey company was 80 percent. A total of 1,830 valid questionnaires (without missing or erroneous entries) were collected, resulting in a total of 10,980 choice experiments. Out of these experiments 3,588 observations corresponding to work trip purpose were selected for the model estimation exercise in this paper. Table 1 summarizes the number of observations chosen for each alternative.

There are two different types of P&R, the first parking closer to the trip origin at the nearest station and the second fringe parking closer to the destination. In the first type, the car is use as an access to the transit mode, while the second type is essentially an auto trip with a kind of remote parking. K&R are usually more of the first type. Travelers' perception may differ between these two types of P&R. The SP experiment was based on the RP data of the travelers' actual trip characteristics. Analysis of the data showed that in our case almost all trips were of the first type were the auto part of the trip was shorter than the public transport part both in terms of time and distance.

Table 1. Number of Chosen Observations in the Sample for Each Alternative

Alternative	Chosen	Percentage (%)
1. Bus with walk access (BW)	1084	30.2
2. Bus with kiss and ride access (BKR)	522	14.5
3. Bus with park and ride access (BPR)	30	0.8
4. Rail with walk access (RW)	850	23.7
5. Rail with kiss and ride access (RKR)	332	9.3
6. Rail with park and ride access (RPR)	31	0.9
7. Rail with bus access (RB)	115	3.2
8. Car driver (CD)	496	13.8
9. Car Passenger (CP)	128	3.6
Total	3588	100.0

This is shown in Figure 1 plotting the cumulative distribution of the proportion of the access time to the total trip time for all transit modes. The median value ranges from 0.1 (Bus with Kiss and Ride access) to 0.36 (Rail with Park and Ride access). This means that in the SP experiment, the respondents were faced with relatively short access times compared to the total trip times.

3.2 Nested Logit Model Specification

It is common with such data sets to estimate two-level NL models as was done by Polydoropoulou and Ben-Akiva (2001), where the assumption is that at the higher level there is public transport or the main public transport modes of bus and rail as well as the car and at the lower level there are the access modes.

An interesting question regarding park and ride and kiss and ride is whether they are considered primarily public transport modes or because the main issue is auto availability are they considered primarily car modes. To investigate this issue and the similarities between the modes, we assume that the nine alternatives are composed of four main modes (nests): Bus, Rail, Car Driver, and Car Passenger.

Note that the main modes above may be further grouped in another nesting level, for example: transit (Bus and Rail nests), and auto (Car Driver and Car Passenger nests). Nevertheless, the limiting assumption of the NL model is that each alternative must be assigned to a single nest.

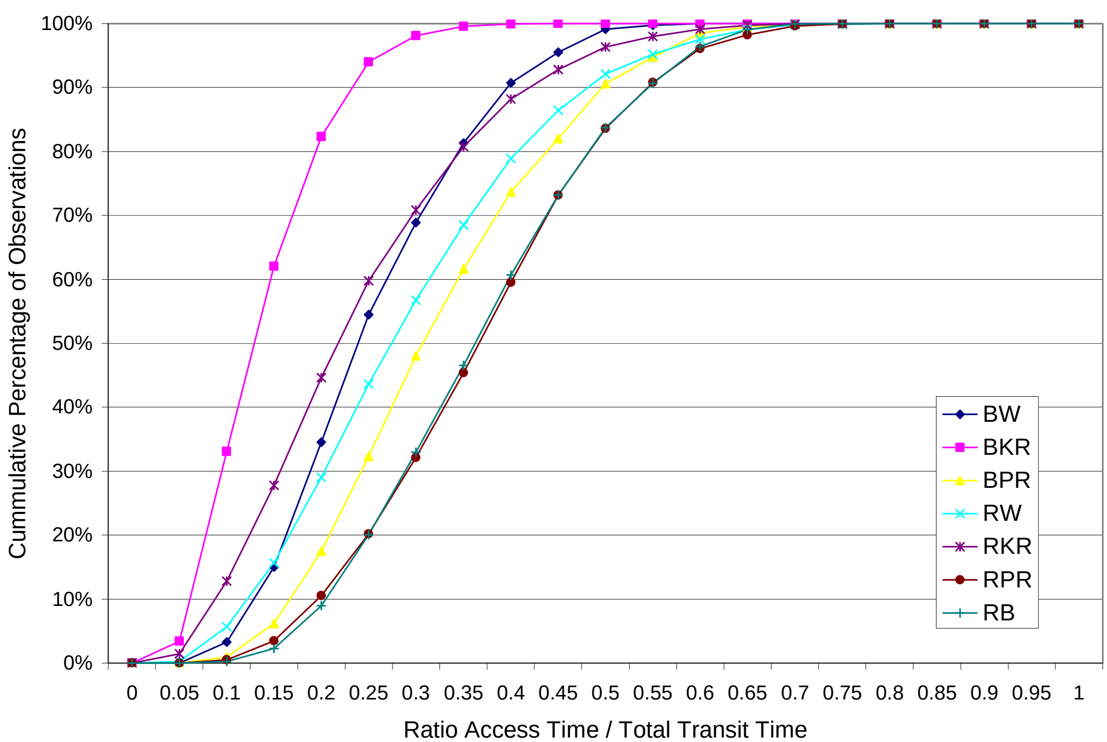
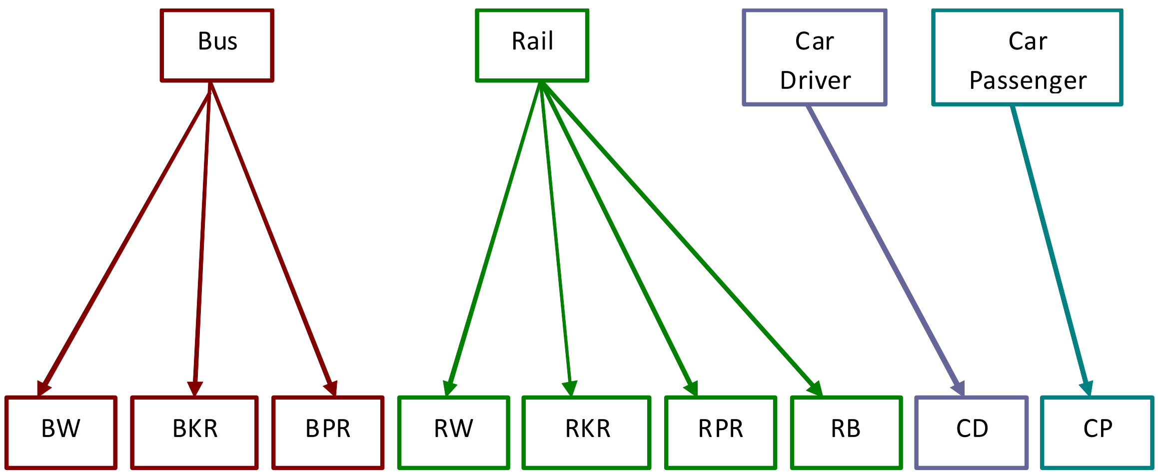


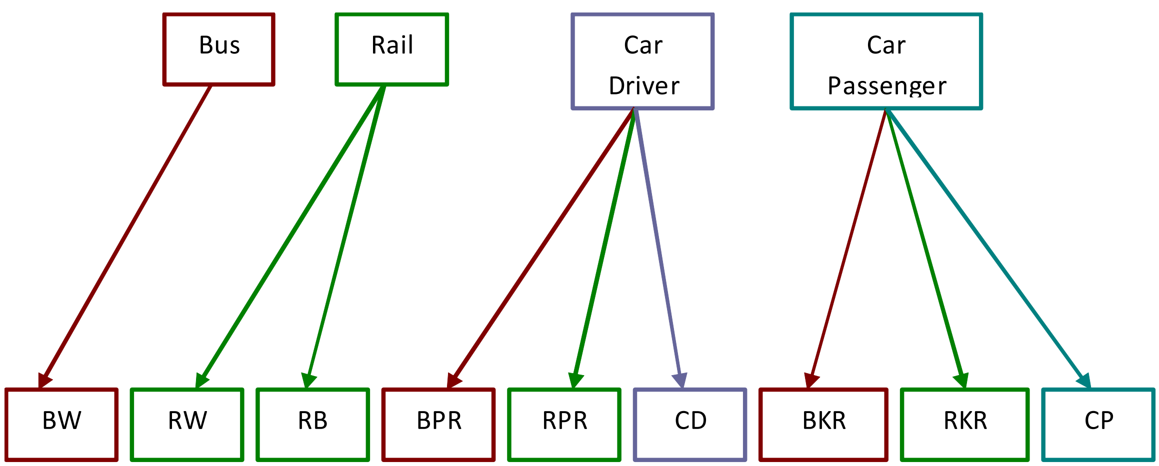
Figure 1. Ratio Access Time to Total Transit Time

We first estimate a simple MNL and two different two-level NL models. The first nested model (NL_transit) has four nests indicated above at the higher level and all the nine modes at the lower level. The park and ride and kiss and ride alternatives belong to the transit nests. The second nested structure (NL_car) assumes that the choice at the higher level is between modes requiring a car including transit with car access versus pure transit modes with either walk or bus access. Figure 2 illustrates the NL structures tested in the paper.

Maximum likelihood estimation was performed using Biogeme (Bierlaire 2003). Table 2 shows the result of the two-level NL models versus the MNL model. These two models were estimated with single nest (logsum) coefficient for all nests.



2a NL_Transit



2b NL_Car

Figure 2. Two Possible NL Structures

Table 2. MNL and NL Estimation Results

Variable	MNL		NL_transit		NL_car	
	coefficient	*t*-stat.	coefficient	*t*-stat.	coefficient	*t*-stat.
Access drive time	-0.048	-5.43	-0.123	-8.80	-0.049	-5.15
Access park cost	-0.003	-5.14	-0.003	-5.37	-0.003	-5.15
Access walk time	-0.159	-19.81	-0.24	-18.06	-0.187	-14.52
BKR constant	-2.387	-21.28	-2.867	-21.18	-2.782	-15.23
BPR constant	-2.421	-8.32	-2.389	-6.83	-2.643	-8.24
CP dummy: 1 Car in hh	-1.76	-7.46	-4.256	-6.25	-1.86	-7.26
KR dummy:1 Car in hh	0.436	4.61	0.433	4.48	0.529	4.40
CD dummy: 2 Cars in hh	0.684	5.22	1.351	4.40	0.845	5.02
CP dummy: 2 Cars in hh	-1.824	-6.26	-4.637	-5.71	-1.86	-5.94
KR dummy: 2 Cars in hh	0.877	7.60	0.885	7.48	1.072	6.88
PR dummy: 2 Cars in hh	0.478	1.76	0.45	1.61	0.663	2.21
CD constant	-1.644	-5.58	-3.503	-5.10	-1.805	-5.13
CP constant	-2.941	-12.29	-5.933	-8.69	-3.385	-11.24
CD cost	-0.001	-3.91	-0.001	-3.35	-0.001	-3.47
CD park search time	-0.057	-3.18	-0.101	-2.82	-0.068	-3.20
CD travel time	-0.064	-10.11	-0.148	-7.31	-0.076	-9.22
CP travel time	-0.061	-7.88	-0.135	-6.33	-0.067	-8.09
CD walk to destination	-0.059	-3.20	-0.112	-3.08	-0.067	-3.08
Transit Fare	-0.003	-14.75	-0.006	-8.80	-0.003	-13.53
Transit In-vehicle time	-0.062	-21.57	-0.138	-10.00	-0.071	-17.41
RB constant	-2.89	-24.92	-3.687	-18.54	-3.164	-19.66
RKR constant	-2.209	-14.96	-2.099	-10.71	-2.569	-12.60
RPR constant	-1.433	-4.50	-0.518	-1.26	-1.578	-4.53
RW constant	0.641	10.30	1.114	8.26	0.764	9.21
Transit Out-vehicle time	-0.063	-12.04	-0.141	-8.08	-0.073	-10.95
Logsum	-	-	0.429	10.47	0.802	15.32
Number of observations	3588		3588		3588	
Number of parameters	25		26		26	
$L(0)$	-6738.4		-6738.4		-6738.4	
$L(\text{final})$	-4978.8		-4935.4		-4973.1	
$\rho(0)$	0.261		0.268		0.262	

The estimation results of the level of service variables exhibit expected signs with magnitude similar to the results obtained by Polydoropoulou and Ben-Akiva (2001). All time and cost coefficients are negative and significant with walk time having the largest absolute value as expected in mode choice models implying expansion service to more stops is the best policy to increase transit ridership. Transit fare is more sensitive than auto cost, suggesting reducing transit fare may attract more riders than increasing parking cost or other auto costs. While some of these coefficient values could be improved in terms of their behavioral appeal, we purposely kept them similar to Polydropoulou and Ben-Akiva (2001) to allow with comparison with their results. We only excluded the parking cost coefficient since it was not significant and did not change the overall results. For further discussion of these results and their policy implications see Polydoropoulou and Ben-Akiva (2001).

Both NL models exhibit a significant improvement of the likelihood with respect to the MNL model. The logsum coefficients in both NL models are significantly lower than 1, indicating that the nested structure better represents the behavioral process.

As can be seen in Table 2, the more conventional NL structure related to transit nests (NL_transit) exhibits a higher log-likelihood value. Therefore, this structure better represents the correlation between the access modes compared to the NL structure related to car nests (NL_car). For this structure we also estimated a model with nest-specific coefficients, one for the bus nest and one for the rail nest. Estimation results are presented in Table 4, together with the CNL model results.

4. The Application of the CNL to Test the Similarities

The idea proposed in this paper is to relate the combined modes to their respective trip legs. Our hypothesis is that park and ride and kiss and ride have some similarities to both auto modes and transit modes.

In specifying the CNL model, we consider the higher level nest to be a choice of one of the four main modes, similar to the NL model structures tested. The lower level nest consists of the nine possible mode alternatives which are actually combinations of these four main modes. Since the focus of this paper is on combined modes such as park and ride and kiss and ride, the CNL model is appropriate to test such a structure. Note that the model can be extended to represent alternatives formed by more than two modes. Figure 3 shows the CNL structure that represents these combinations indicating which branch each allocation parameter is estimated for.

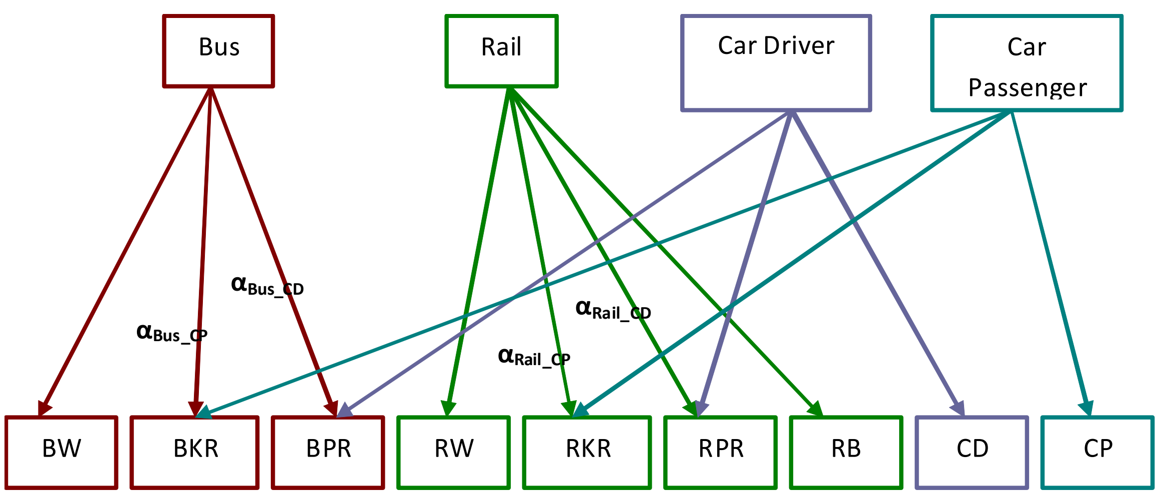


Figure 3. Structure of the Cross-Nested Logit Model

Each alternative is formed by either one main mode (in this case, the alternative belongs to a single nest) or two main modes (in this case, the alternative belongs to two nests). For example, the RPR alternative belongs to the "Rail" and the "Car Driver" nests, and the CP alternative belongs only to the "Car Passenger" nest. According to this idea, the RB alternative should belong to the "Rail" and "Bus" nests, but this structure did not produce satisfactory results, therefore the RB alternative was assigned to the "Rail" nest only.

The nesting structure can be also represented by a mode-nest incidence matrix, as presented in Table 3. According to this specific model specification, there are 4 coefficients to be estimated. The blank entries in Table 3 indicate that the alternative does not belong to the nest.

4.1 CNL Estimation Results

Several CNL specifications were tested. Table 4 summarizes the results for two selected CNL models, together with the nest-specific NL_transit model. The first CNL model has a single logsum coefficient, and the second CNL model has two nest-specific coefficients for the bus and rail nests, similar to the NL_transit model. In this model, the nest coefficients for car driver and car passenger nests were constrained to 1. The reason for this constraint is related to the fact that the best NL model found (NL_transit) has simple nesting structures for car driver and car passenger alternatives. For convenience of the analysis, only nesting and inclusion coefficients are presented, since the explanatory variables have coefficient estimates similar to the NL_transit model.

Table 3. Cross-Nested Logit Model – Inclusion Coefficients

	Nest			
Alternative	Bus	Rail	Car Driver	Car Passenger
BW	1			
BKR	α_{Bus_CP}			$1-\alpha_{Bus\_CP}$
BPR	α_{Bus_CD}		$1-\alpha_{Bus\_CD}$	
RW		1		
RKR		α_{Rail_CP}		$1-\alpha_{Rail\_CP}$
RPR		α_{Rail_CD}	$1-\alpha_{Rail\_CD}$	
RB		1		
CD			1	
CP				1

Table 4. Selected NL and CNL Estimation Results

Variable	NL_transit - Nest-specific coefficients		CNL - Single nest coefficient		CNL – Nest - specific coefficients	
	coefficient	*t*-stat.*	coefficient	*t*-stat.*	coefficient	*t*-stat.*
Logsum	-	-	0.423	9.87 (-13.48)	-	-
Logsum (bus nest)	0.412	7.71 (-4.53)	-	-	0.401	4.41 (-2.64)
Logsum (rail nest)	0.433	10.11 (-5.73)	-	-	0.398	9.60 (-5.78)
α_{Bus_CP}	-	-	0.988	52.0 (-0.61)	0.852	4.59 (-0.79)
α_{Bus_CD}	-	-	1	Fixed	1	Fixed
α_{Rail_CP}	-	-	0.981	28.0 (-0.54)	0.563	3.25 (-2.52)
α_{Rail_CD}	-	-	1	Fixed	1	Fixed
Number of observations	3588		3588		3588	
Number of parameters	27		28		29	
$L(0)$	-6738.4		-6738.4		-6738.4	
L(final)	-4934.3		-4935.4		-4930.9	
$\rho(0)$	0.269		0.268		0.269	

* *t*-statistics with respect to 0 (and with respect to 1 in parenthesis)

Both CNL models presented in Table 4 were estimated fixing two coefficients related to park and ride (α_{Bus_CD} and α_{Rail_CD}). The reason for fixing the coefficients was that estimation results yielded values very close to the pre-specified limit of 1, causing some numerical problems. This means that BPR and RPR belong respectively to the Bus and Rail nests, as in the NL model. Note that there are relatively few observations related to park and ride, and this may explain the reason for not finding significant correlations between these nests.

The best CNL model was found including nest-specific coefficients for the bus and rail nests (the last model in Table 4). Note that in this model, the inclusion coefficient α_{Bus_CP} (Kiss and Ride for Bus nest) is significantly different from zero, but not significantly different from 1. The only inclusion coefficient that is significantly different from 1 is the coefficient α_{Rail_CP} (Kiss and Ride for Rail nest). The likelihood ratio test of this model indicates that this CNL model outperforms the NL_transit model with the nest-specific coefficients at the five percent confidence level. However, since most inclusion coefficients are either one or zero, the CNL model is essentially very similar to the NL_transit model.

It should be noted that according to Polydopoulou and Ben-Akiva (2001), the NL_transit model (Figure 2a) was reported as the best NL model obtained for this SP dataset. In this structure, Car Passenger and Car Driver belong to different nests, and in the CNL model these two nests were also kept apart. Additional CNL structures were tested, but not produce significantly better results.

In line with the interpretation mentioned above, the estimated inclusion coefficients for the combined alternatives of car and transit modes show that the combined alternatives are primarily related to their respective transit nest (either bus or rail), and only marginally related to their respective car nest (either driver of passenger). Therefore, in the dataset used for this case study, the transit alternatives do not correlate much with the car alternatives.

5. The Application of the Logit Kernel to Test the Similarities

The key issue in applying the factor-analytic Logit Kernel (LK) to our mode choice problem is the specification of the covariance matrix of the Probit-type random error terms. In practice, this requires the specification of the elements of the **F** and **T** matrices. There can be various options to represent the similarities among the alternative modes and therefore there is a need to make some additional assumptions regarding the covariance matrix.

First, we assume that the mode specific factors are i.i.d. Normal. Second, the **F** matrix is an incidence matrix (0/1 matrix) that connects each alternative (the rows) to its corresponding nest (the columns), similar to Table 3 presented for the CNL case. Third, the **T** matrix is the nest factors variance matrix, in this case a diagonal matrix. Finally, we further assume that there is a unique variance for each nest. The **T** matrix is then obtained as follows:

$$\mathbf{T} = \begin{bmatrix} \sigma_{bus} & 0 & 0 & 0 \\ 0 & \sigma_{rail} & 0 & 0 \\ 0 & 0 & \sigma_{cd} & 0 \\ 0 & 0 & 0 & \sigma_{cp} \end{bmatrix} \tag{11}$$

The **T** matrix above has only diagonal elements. This allows quick computation of the utility vector and covariance matrix. The resulting covariance matrix **FTT'F'** of the Probit-type random error elements is shown in Table 5. As can be seen from Table 5, the covariance of the error terms between each two alternatives equals to the variance of the nest that these two alternatives share. For example, covariance between bus with walk access (BW) and bus with kiss and ride access (BKR) is σ_{bus}^2, which corresponds to the Bus nest. The variance of each alternative (the diagonal) is the sum of the variances of the nests that compose it. For example, the variance of bus with park and ride access (BPR) is $\sigma_{bus}^2+\sigma_{cp}^2$.

Table 5. Covariance Matrix of the Probit-type Random Error Elements

	BW	BKR	BPR	RW	RKR	RPR	RB	CD	CP
BW	σ_{bus}^2	σ_{bus}^2	σ_{bus}^2	0	0	0	0	0	0
BKR	σ_{bus}^2	$\sigma_{bus}^2+\sigma_{cp}^2$	σ_{bus}^2	0	σ_{cp}^2	0	0	0	σ_{cp}^2
BPR	σ_{bus}^2	σ_{bus}^2	$\sigma_{bus}^2+\sigma_{cd}^2$	0	0	σ_{cd}^2	0	σ_{cd}^2	0
RW	0	0	0	σ_{rail}^2	σ_{rail}^2	σ_{rail}^2	σ_{rail}^2	0	0
RKR	0	σ_{cp}^2	0	σ_{rail}^2	$\sigma_{rail}^2+\sigma_{cp}^2$	σ_{rail}^2	σ_{rail}^2	0	σ_{cp}^2
RPR	0	0	σ_{cd}^2	σ_{rail}^2	σ_{rail}^2	$\sigma_{rail}^2+\sigma_{cd}^2$	σ_{rail}^2	σ_{cd}^2	0
RB	0	0	0	σ_{rail}^2	σ_{rail}^2	σ_{rail}^2	σ_{rail}^2	0	0
CD	0	0	σ_{cd}^2	0	0	σ_{cd}^2	0	σ_{cd}^2	0
CP	0	σ_{cp}^2	0	0	σ_{cp}^2	0	0	0	σ_{cp}^2

The Utility vector is then obtained as follows:

$$U = \begin{bmatrix} \beta X_1 + \sigma_{bus}\zeta_1 + \nu_1 \\ \beta X_2 + \sigma_{bus}\zeta_2 + \sigma_{cp}\zeta_3 + \nu_2 \\ \beta X_3 + \sigma_{bus}\zeta_2 + \sigma_{cd}\zeta_4 + \nu_3 \\ \beta X_4 + \sigma_{rail}\zeta_4 + \nu_4 \\ \beta X_5 + \sigma_{rail}\zeta_2 + \sigma_{cp}\zeta_4 + \nu_5 \\ \beta X_6 + \sigma_{rail}\zeta_2 + \sigma_{cd}\zeta_4 + \nu_6 \\ \beta X_7 + \sigma_{rail}\zeta_4 + \nu_7 \\ \beta X_8 + \sigma_{cd}\zeta_2 + \nu_8 \\ \beta X_9 + \sigma_{cp}\zeta_2 + \nu_9 \end{bmatrix} \qquad (12)$$

Note that both the LK and CNL models were specified with the same explanatory variables, and 4 additional parameters. However, in each model these parameters indicate different correlation options, as illustrated in Table 3 (CNL) and Table 5 (LK).

These examples emphasize and question some of the assumptions we made above and illustrate the need to test other specifications as well. For example, we assumed that the covariance between the bus alternatives is the same for any two modes that share the bus mode. In other words, the covariance between bus with walk access (BW) and bus with kiss and ride access (BKR) is the same as the covariance between bus with walk access (BW) and bus with park and ride access (BPR). As long as two modes share the bus mode the covariance among them is σ_{bus}^2. An alternative approach would be to assume a different variance for each bus alternative. In an effort to keep a relatively small and tractable number of parameters to estimate we assume in this paper that they are the same.

The main question of interest in this paper is the similarity between transit modes that share the car mode. We assumed a distinction between car driver and car passenger and for each one of them we assume a different variance. However, the covariance between modes who share the car as access mode such as the rail with park and ride and the bus with park and ride is similar to the covariance between modes who share the car as either access or main mode, such as the rail with park and ride mode and the car driver mode. Note that this approach is similar to the approach suggested in the CNL model.

We are specifically interested in the estimated variance of the within car driver mode (σ_{cd}^2) and of the within car passenger mode (σ_{cp}^2). A significant variance here will indicate that there are unobserved similarities between car as main mode alternatives and between public transport alternatives with car access. This means that the simple NL model type will not represent well the choice problem, since it assumes that the park and ride alternative share unobserved characteristics with the other public transport modes but not with the car modes.

Another alternative specification to test could be that the variances of the unobserved characteristics of the car driver and car passenger are similar, i.e., constrain σ_{cd}^2 to be equal to σ_{cp}^2. In addition, we assume no covariance between modes that share public transport of different technology, i.e., between bus and rail, and we assume no covariance between car driver and car passenger. This is only a partial list of alternative specifications that could be tested; we limit ourselves in this paper to a relatively simple specification as a first test.

5.1 LK Estimation Results

Table 6 shows three LK estimation results, one with the MNL as kernel, the second with NL as kernel (similar to NL_transit) and the third with CNL as kernel. As with the CNL model results, only the additional coefficients are presented in the table, since the explanatory variables have coefficient estimates similar to the respective MNL and NL_transit models. The LK estimation results were performed with 1,000 Halton draws, which produced stable results when comparing with estimation runs for fewer draws.

After performing several trials with different LK specifications for the factor analytic terms, the coefficients σ_{bus} and σ_{rail} were found not significant, showing no additional common error term within the rail and bus models in additional to those that are presented by the nested logit kernel of the model, and therefore were omitted in Table 6.

Note that these two coefficients respectively indicate correlations within the Bus and Rail nests. Similar to the CNL model, a significant correlation between the access modes and the main transit mode was not found.

Table 6. Selected LK Estimation Results

Variable	LK with MNL		LK with NL_transit		LK with CNL	
	Coefficient	*t*-stat.	Coefficient	*t*-stat.	Coefficient	*t*-stat.
Logsum (bus nest)	-	-	0.318	3.97	0.150	3.43
Logsum (rail nest)	-	-	0.412	9.23	0.398	9.39
σ_{cd}	1.413	3.98	0.931	2.30	1.020	3.93
σ_{cp}	-1.248	-3.51	-1.554	-3.36	-0.483	-2.94
α_{Bus_CP}	-	-	-	-	0.681	8.84 (-4.14)
α_{Bus_CD}	-	-	-	-	1	Fixed
α_{Rail_CP}	-	-	-	-	0.763	4.61 (-1.43)
α_{Rail_CD}	-	-	-	-	1	Fixed
Number of observations	3588		3588		3588	
Number of parameters	27		29		31	
L(zero)	-6738.4		-6738.4		-6738.4	
L(final)	-4971		-4928.6		-4924.8	
Rho (zero)	0.261		0.268		0.269	

The coefficients of the 25 explanatory variables (not presented in the table) are similar in terms of their coefficient and t-statistic magnitudes to the respective MNL and NL models presented in Table 2 and are not significantly different from them. The coefficients σ_{cd} and σ_{cp}, respectively related to park and ride and kiss and ride, were found significant in all the LK models. The LK with NL model significantly improves the likelihood of the LK with MNL model showing that there are significant unobserved similarities among the different transit modes. Note that LK with NL is also significantly better than the correspondent NL model at a one percent significance level. This means that there are also some unobserved similarities between the auto modes and transit modes that use auto access. The likelihood of the LK with MNL model is also lower than the NL model. This means that the unobserved similarities among the various transit modes are stronger than the similarities between auto and transit modes with auto access. The best likelihood is achieved for the LK with CNL as a kernel which is significantly better than the CNL showing the advantage of this most flexible form to capture the complex similarity relations between all these modes.

In all the LK models, both coefficients σ_{cd} and σ_{cp} were significant, in contrast to the CNL model, that only exhibited a single significant coefficient. This might be related to the more flexible structure of the LK model, in comparison to the CNL model. Note that these two coefficients indicate correlations between a transit mode and a car mode, similar to the coefficient in the CNL model. In the LK with CNL kernel, the most flexible form, there are three significant coefficients showing the complex similarity among these various modes. The significant σ_{cd} and σ_{cp} show the correlation between transit and car modes, and the CNL kernel shows significant joint inclusion for the bus with car passenger access to both modes and non-significant joint inclusion for the rail with car-passenger access to both modes. However, in both cases the inclusion coefficients are larger than half showing a stronger inclusion to the transit nest, and α_{Bus_CD} and α_{Rail_CD} had to be fixed to one showing a NL transit like structure. In other words, overall the CNL kernel shows that these mixed modes belong more to the transit nest.

6. Summary

This paper proposes an approach to test similarities among modes and the appropriate model structure, providing the flexibility for various model structures including CNL and LK. The approach is applicable to model various situations of simultaneous choices in transportation such as the simultaneous decision regarding mode and time of travel, mode and destination and others more complicated travel choice. With the move toward activity-based modeling it is important to provide the econometric tools to adequately model such complicated decisions.

In this paper, both the CNL and LK alternatives were grouped to nests according to the pure modes. This means that a combined mode such as RPR was assumed to belong to "Rail" and "Car Driver" nests. Following the same idea, the CD and CP alternatives could be further decomposed to belong respectively to "Car" and "Driver" nests and "Car" and "Passenger" nests, and so on. The paper presented the best estimation results with the available SP dataset for a given nesting structure.

Table 7 compares the final log-likelihood results for all estimated models and the number of parameters estimated for each model. The models are sorted by increased order of generality, in the sense that each model is a special case of a more general model that appears further down in the table. The second column of the table indicates the specific model that each model generalizes.

Table 7. Summary of Estimation Results

Model	Generalization of model	Final Log-Likelihood	Number of Estimated Parameters
0. Null	-	-6738.4	0
1. MNL	-	-4978.8	25
2. NL_car (single nest coefficient)	MNL	-4973.1	26
3. NL_transit (single nest coefficient)	MNL	-4935.4	26
4.NL_transit(nest-specific coefficients)	MNL	-4934.3	27
5. CNL (single nest coefficient)	NL_transit (3)	-4935.4	28
6. CNL (nest-specific coefficients)	NL_transit (4)	-4930.9	29
7. LK (MNL as Kernel)	MNL	-4971.0	27
7.LK (NL_transit single nest as Kernel)	NL_transit (3)	-4928.9	28
8.LK(NL_transit nest-specific as Kernel)	NL_transit (4)	-4928.6	29
9. LK (CNL single nest as Kernel)	CNL (5)	-4927.3	30
10. LK (CNL nest-specific as Kernel)	CNL (6)	-4924.8	31

The results presented in the table are consistent and expected, that is, the more general models always exhibit a higher log-likelihood compared to the specific case model.

Specifically, we can compare models 1 to 4 (MNL and NL models); then compare models 1 – 3 – 5 (MNL-NL-CNL) or 1 – 4 – 6 (MNL-NL-CNL nest-specific options); then compare 1 to 7, 3 to 8, 5 to 9, and 6 to 10 (the Mixed Logit versions of MNL, NL, and CNL models, specifically).

It is interesting that in the dataset used for model estimation, the flexibility provided by the CNL model has almost collapsed back to NL model with the traditional structure, where the various transit modes are at the higher level of the tree and the various access modes at the lower level of the tree. This indicates that there are more unobserved similarities among the various transit modes with different access mode than among the various modes using auto either as a main mode or as an access mode. We had expected that with superior transit mode such as the rail alternatives there would be more unobserved similarity among modes sharing the use of auto, as one of the main issues in mode choice is auto availability making the choice of auto use a higher level choice, and then whether you park and ride or drive all the way a lower level choice.

The results in this paper agree with the traditional thinking that park and ride is mainly associated as a transit mode. However, this could also be explained by the fact that the SP survey was conducted among people not familiar with premium transit modes and therefore not used to park and ride. In a recent survey of public transit users in Tel Aviv, it was found that on average less than half percent of them had a car available for their trip, meaning that those who have car don't use public transport and therefore there is a very limited choice of park and ride mode. This is different in corridors with good rail service, like the coastal corridor (Haifa – Tel Aviv), where 18 percent of the rail riders had access to a car. However, there is no currently such service in the new mass transit corridor where the SP survey was conducted.

This paper specified the factor analytic LK to mode choice situation. The key issue was the assumption of a diagonal factor analytic matrix, which allowed the computation of the covariance matrix at affordable computer resources. The estimation results showed that all LK models had significantly better likelihood than their specific cases, the MNL , NL and CNL models (model 10 is better than model 6, model 9 is better than model 5, model 8 is better than model 4, and model 7 is better than model 3).

The best likelihood was achieved for the LK with CNL kernel which is the most general and flexible form thus best revealing the complex similarity among transit with car access modes to both transit and car. Overall the significant co-variances of the LK part show the correlation between the transit and car modes, while the CNL kernel shows that overall most transit with car access modes belong more to transit than to car modes.

Note that the LK model can be extended in several other ways, for example, capturing taste variation. This paper purposely focused on the issue of capturing similarity between auto and transit alternatives, and therefore other model specifications that capture other similarity types were not included in this paper.

Along with the need of further research on LK model estimation issues, there is a need to test the methodology proposed in this paper using other datasets, to check if the results obtained in this paper reflects only local behavior or the phenomenon is more general. Other more flexible model structures should also be tested to capture such similarities.

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