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A congestion-dependent, Dynamic Flexibility Model of freeway networks

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ABSTRACT

The concept of flexibility is very common in many engineering aspects but less common in transportation planning. This study develops a dynamic model of system flexibility for freeway networks. When comparing different freeway networks, it is essential to have a measure defining a network's flexibility based not only on its topology but also on the amount of traffic compared to each segment's capacity. Application of the proposed flexibility model could be helpful in estimating the perceived amount of total system congestion in a given network at any given time and how this congestion may vary if a new link is added to the system. The proposed Dynamic Flexibility Model is based on three variables: the number of routes between each origin–destination (O–D) pair in the freeway network, the amount of length in common in each O–D pair, and the amount of variability among the various routes in each O–D pair relative to the shortest path in the same O–D. The traffic flow is included in the model via the perceived length: the relevant length is equal to its actual length in an uncongested state and increases during periods of congestion, since drivers then experience longer travel times, which are converted into an equivalent increase in the perceived-segment length. In this work, the onset of congestion that was adapted was at the Critical Occupancy Point (COP), which is an objective measure evaluating whether or not a freeway segment is congested. This paper shows that the proposed Dynamic Flexibility Model is able to compare different freeway networks, thus adding a new dimension for measuring drivers' flexibility in choosing other routes in case some links suffer congestion and breakdown. Furthermore, the proposed model helps to define the perceived total congestion in the system.

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1. Introduction

A system of freeways can be described as a network of nodes or vertices connected by links.

When looking at freeway networks, previous studies mainly considered the disruption of links as being due to major catastrophic events or abnormal behavior during a disaster (Fang and Wakabayashi, 2012). That is, investigators examined what happens to network flow when a link is completely unusable; hence, their focus was on network robustness and reliability. One definition of a robust network is a network that can compensate for disruptions in its links with no more than a small increase in the overall travel time in the system (Miller-Hooks et al., 2012). The models that are used in existing research capture the importance of each link to the overall performance of the network. When one link is congested, the congestion

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may potentially impact the entire network. The disruption of a link does not necessarily cause its capacity to drop to zero. The state of a link, indicating whether it is congested or not, is determined in this study by a threshold value of occupancy, termed the Critical Occupancy Point (COP). The flexibility model that is developed is dynamic in nature because the state of each link changes continuously over time.

This model, on the one hand, can define the flexibility of a theoretically "empty" network and thus actually provide a benchmark based on the network's topology; on the other hand, it can calculate the flexibility of the system with a given amount of traffic. The ratio between the two results defines the amount of perceived congestion in the system or "perceived network congestion."

Previous studies have not viewed the flexibility of a freeway network as a dynamic issue. Therefore, the objective of this study was to define a Dynamic Flexibility Model that enabled its users to evaluate system congestion and system flexibility (under congestion conditions) and to compare several freeway networks for their congestion and flexibility, thus enabling the development of measures to increase flexibility and reduce congestion.

2. Previous studies

Network design problems (NDP) deals with "road network reliability," which previous studies defined in different ways. Capacity reliability considers the uncertainties associated with arc capacities, as Chen et al. (2000) explained, "when the roadway capacities are assumed to take only discrete binary values (zero for total failure and one for operating at ideal capacity), then capacity reliability includes connectivity reliability as a special case." Researchers have variously related it to connectivity reliability, which implies the probability that network nodes remain connected (i.e., that there exists a path between the nodes, with some links operating at full capacity and others in complete failure; Iida and Wakabayashi, 1989); to travel time reliability, which is associated with the probability that a trip between a given O–D can be made within a specific interval of time (Asakura and Kashiwadani, 1995); and to capacity reliability, which is the probability that the network can accommodate a certain traffic demand at a required service level (Chen et al., 2000).

Fang and Wakabayashi (2012) proposed a model to calculate the reliability and importance of links in a network during a disaster event. An example of previous definitions of network flexibility can be seen in the study by Feitelson and Salomon (2000), who conducted a conceptual discussion of network flexibility, which they defined as "the ease with which a network can adjust to changing circumstances and demands, both in terms of infrastructure and operation." They divided the concept into three separate terms: node flexibility – the ease with which network nodes can be sited; link flexibility – the ease and cost of inserting an additional link between nodes; and temporal flexibility – the ability to sequence transportation investments and the degree to which this transportation requires coordination among users (so that the link's use by one user does not prevent its use by others).

Morlok and Chang (2004) defined flexibility of transportation systems as "the ability of a transport system to accommodate variations or changes in traffic demand while maintaining a satisfactory level of performance." They studied flexibility in respect to changes in demand or traffic, which was defined as network capacity; employing linear programming, they resolved the issue of the maximization of a system's total cargo traffic, subject to routing and resource constraints. Sun et al. (2006) extended Morlok and Chang's (2004) capacity-flexibility model to include the uncertainty of traffic patterns, or volume–delay functions; they added a stochastic traffic-assignment procedure to eliminate the need for path enumeration and to make the model more useful for large networks.

Miller-Hooks et al. (2012) defined a "network resilience measurement tool" that incorporated preparedness and post-disaster recovery actions, as well as the potential impact of those actions. Network resilience, α, presented in Eq. (1), was defined as the expected fraction of demand that can be satisfied post-disaster.

$$\alpha = E\left(\frac{\sum_{w \in W} d_w}{\sum_{w \in W} D_w}\right) \tag{1}$$

where D_w is the original pre-disaster demand for O–D pair w; d_w is the post-disaster maximum demand that can be satisfied for O–D pair w.

The first attempt to calculate and assign a certain value to the flexibility of a transportation system was undertaken by Scott et al. (2006), who pointed out that the volume/capacity (v/c) ratio does not always indicate which highway segment is the most critical. They offered an approach for identifying critical links and evaluating network performance that they called the Network Robustness Index (NRI). NRI calculates the weight of each link in a given network as the additional travel time resulting from the link's being removed. In other words, the NRI of Link A is the travel time in the equilibrium assignment without Link A minus that with Link A. In this way, the individual importance of each link is obtained. Scott et al. also calculated the average index of all links in the network and compared it to different networks. Sullivan et al. (2010) extended NRI by proposing the Network Trip Robustness (NTR) measure, which allows different networks to be compared. It is calculated as the sum of all NRIs for all links in the network divided by the total demand for all O–D's in the network. Thus, the NTR metric can be used to compare different size networks with differing levels of connectivity and varying demand.

When discussing networks, Cascetta et al. (1996) mentioned the term "commonality factor" and presented a method for reducing the utility of overlapping routes. This reduction in overlapping routes was presented in a study by Zhou et al.

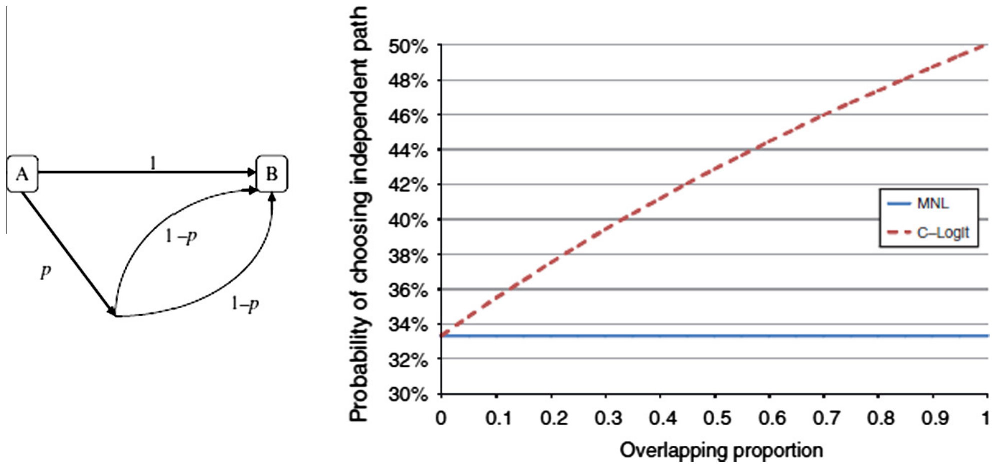

Fig. 1. Probability of choosing the independent route (taken from Zhou et al., 2010).

(2010), who investigated the stochastic user equilibrium problem with the route choice model based on the C-logit function. The network loading of a network with a common link is presented in Fig. 1 in this study.

There are three routes with a total length of 1 unit, one route being independent and the two others having a common link the length of p ($0 \leq p \leq 1$). The figure presents the probability of choosing the independent route under the Multinomial Logit (MNL) model or under the C-logit model proposed by Cascetta et al. (1996), taking into account mutual lengths of the three routes. The longer the mutual link, the higher the probability of one's choosing the independent route.

Network congestion of freeways was studied by Polus (1996), who proposed to measure it as the sum of all Minute Miles of Congestion (MMC) in the sections composing the network. MMC is calculated as the sum of the products of congestion duration multiplied by the congestion physical length; the measure was estimated for all expressways the amount and duration of congestion on a given expressway included in the network. Another proposed way to calculate network congestion was the Mobility Monitoring Program (MMP), studied by Turner et al. (2004) which is an area-wide Travel Time Index (TTI). MMP calculates the TTI for each freeway as the ratio of average peak travel time to a free-flow travel time and then defines the average TTI of all the freeways composing the network through weighting each freeway section by the respective Vehicle Mile Traveled (VMT) of this section.

Previous studies investigated the importance of each link separately, as Scott et al. (2006) did, or calculated the resilience of a network in the case of reduced capacity on certain links, as Miller-Hooks et al. (2012) did. No study up to now has calculated the flexibility of a network based on dynamic conditions and a congestion state that may be present at specific times on certain links. Following the definition of NTR proposed by Sullivan et al. (2010), and taking into account that demand can vary throughout the day, the flexibility model proposed in this study is a dynamic model that depends on the prevailing flow conditions. The ideal, most flexible network that can be imagined is one that has infinity routes between each O–D, all routes being independent and equal in length. Therefore, the flexibility model is dependent on the number of effective routes (i.e., the relevant, congested-dependent routes), the amount of common lengths, and the degree of variability among the effective freeway routes. The proposed flexibility model adds a new dimension to network congestion that may be helpful in quantifying the amount of perceived congestion in a given system and, thus, in furthering economic analyses or improving comparisons of several networks.

One possible way to estimate perceived network congestion is to calculate the ratio of the flexibility of a ''loaded'' network (i.e., with traffic) to an ''empty'' network (i.e., without traffic or with low volumes). Note that an ''empty'' network could mean a network with low volumes and no congestion.

3. Perceived lengths

In order to develop a dynamic model for the flexibility of a highway network, given a known origin and a known destination, it was decided initially to use the lengths of the links composing the routes in the network. A link in this study is between two nodes that represent every change in traffic: when traffic enters/exits, there might be a change in its volume and, therefore, a change in the existence of congestion.

The perceived length of a link in the freeway network depends on the following characteristics:

1. physical length of each link;
2. presence of congestion;
3. amount of traffic compared to the segment's capacity, which may be measured by various traffic variables, such as the volume-to-capacity ratio.

The perceived length of each link is equal to its absolute length only if there is no congestion on the link. Otherwise, when the link is in a congested state, the travel time that drivers experience is longer than it would be in a non-congested situation. As the time to pass a link increases, so does its ''perceived'' length in comparison with its absolute length. The threshold used in this research in order to define the onset and duration of congestion is based on the Critical Occupancy Point (COP) according to Sofer et al. (2012). This measure combines two kinds of parameters, speed and occupancy, and may be developed from data obtained independently from each freeway sensor because the information on each sensor provides measurements that reflect the flow conditions at that particular location. Note that there are alternative definitions for the critical point, based on the fundamental diagram (e.g., Jin and Ran, 2009). COP is the inflection point in the relationship of speed as a function of occupancy; i.e., the point at which the function changes from a concave to a convex function; this phenomenon is described in Eq. (2).

$$f = S_{\min} + \frac{S_{FFS} - S_{\min}}{1 + e^{\beta(\mathrm{OCC} - \mathrm{COP})}} \tag{2}$$

where f is the curve-fitted speed; S_{FFS} is free-flow speed (FFS), calculated from the average low-volume speeds detected by the sensor; $S_{\min}$ is minimum speed calculated by the sensor for the given observations; β is parameter describing the steepness of the function; OCC is occupancy recorded by the sensor; COP is the Critical Occupancy Point.

Prior to the COP, the state of flow is un-congested, and then even a small decrease in speed relative to the FFS does not cause congestion. Drivers are not likely to be sensitive to this small increase in speed. However, beyond the congested flow state, not only is the speed reduced significantly, but the flow is also unstable (stop and go, with substantial fluctuations among small speeds). Our model proposes that only in a congested state in order to capture the unstable flow it is needed to increase the lengths of the links.

Fig. 2 shows the relationship between volume and capacity and also the ratio of observed speed to free-flow speed, but only for congested observations; i.e., when the observation from one sensor on the Ayalon Freeway was in the convex area in Eq. (2).

Eq. (3) describes the shape of the model in Fig. 2.

$$S_{\mathrm{observed}} = S_{FFS}\left(a + \alpha\left(\frac{v}{c}\right)^{\theta}\right) \tag{3}$$

where α, θ, a is the parameters estimated from the observations; S_{observed} is observed speed detected by the sensor; S_{FFS} is FFS calculated from the average low-volume speeds detected by the sensor; v is volume detected by the sensor; c is the capacity of the freeway.

The fitted curve is similar to the function of the Bureau of Public Roads (as it appears in Skabardonis and Dowling, 1997), which calculates the time it takes to pass a link as a function of the volume on this link. Eq. (3) assumes that the capacity is constant at all times and that v/c is limited to the maximum level of 1. The speed is divided by the constant FFS on the Y axis, and the volume by the capacity on the X axis for all unstable-flow data points. The parameters a, α and θ in Eq. (3) are 0.1358, 0.4155 and 3.4806, respectively. This estimation is based on the best-fit technique applied to observations of congestion taken at 5-min periods from 13 sensors and 5 data sets at each sensor.

Perceived length reflects the amount of time that it takes to traverse a link. In cases in which the links are not congested, the perceived lengths are equal to the absolute lengths, along which travel is at a free-flow-speed. In contrast, the travel-time along a congested link increases. Hence, perceived length is a function of the v/c for that link; the more severe the congestion, the longer is the perceived length. The length of Link j is given by Eq. (4).

$$k_j = \begin{cases} \frac{l_j}{a+\alpha\left(\frac{v}{c}\right)^{\theta}}, & \text{if link j is congested} \\ l_j, & \text{if link } j \text{ is not congested} \end{cases}$$

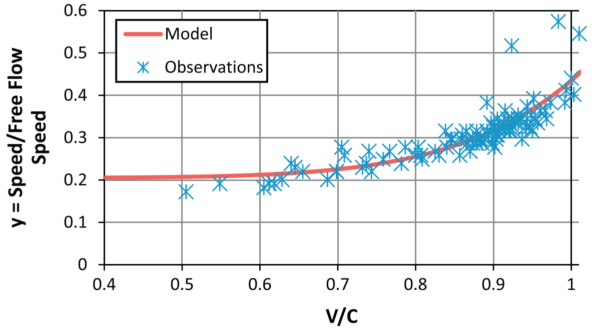

Fig. 2. Ratio of speed/free-flow speed to v/c for all congested observations, 5-min average data.

where l_j is the absolute length of Link j; k_j is relative length of Link j (i.e., the perceived length); $\alpha,\ \theta,\ a$ is the parameters estimated from the observations.

All links discussed in this work are commonly used and relevant. Links that are not reasonable were not considered.

4. Flexibility model

Once the perceived lengths of all links are calculated, the model will yield a single value of network flexibility for a given freeway system at a certain time of day and under specified flows for a given O–D. The model developed, which is presented below, has the following properties:

1. The model is consistent for all combinations of schematic networks examined; that is, it yields higher flexibility values for networks with more routes between pairs of origin–destinations than for networks with fewer routes, because the former enables more route options between different O–Ds.
2. The model is sensitive to the lengths of all possible, relevant, high-capacity routes (freeways and toll roads) in the network; it takes into account the existence of congestion on the different links.
3. The model yields flexibility values ranging between 0 and less than 1; a network with only one route results in 0 flexibility, while a network with infinite independent routes yields a flexibility approaching 1.
4. The model results in higher values for networks whose alternative routes are similar in length relative to the shortest route between each O–D; that is, when overall variability among alternative routes between each O–D is minimal, the mode results in a value of flexibility nearer to 1.
5. The model provides higher values to networks whose alternative routes have fewer lengths in common with all other routes. The reason is that when the common value increases, flexibility decreases, as the probability of a single-breakdown chaotic impact increases.

Requirements 1 and 4 take into account the availability of other routes if the freeway is temporarily blocked or congested. As Cohen and Southworth (1999) suggested, accidents may cause queue formation. Queues in our developments correspond to congestion that occurs when volume is larger than capacity. Thus, the longer the links that are not unique (common links) to an origin–destination pair, the higher will be the probability of random congestion, and therefore these links should be avoided. The probability of congestion is even larger for longer links that serve several routes, since Poisson intensity also increases with traffic volume. Cohen and Southworth (1999) found that the occurrence of incidents on a highway section is governed by a Poisson process that depends on incident rate, volume, and section length. Therefore, avoiding common distances with other routes as far as possible increases the probability that an alternative non-congested route can be found even if a random breakdown occurs at some location along other routes.

The model developed depends on the perceived lengths as calculated in Eq. (4) and on three influencing variables according to the ideal, most flexible network that can be imagined:

1. number of routes between a given O–D – the more routes, the more flexible the network system;
2. common lengths among all routes, based on the commonality factor;
3. amount of variability among available freeway routes relative to the shortest route between each O–D.

All three flexibility-impacting variables need to be included in the model, yet each one has a significant impact on its own. In order to obtain the best measure, a product function would provide a better estimation than would an additive function, where one variable could irrationally bias the result. In a product, all variables contribute simultaneously to the measure, thus producing an unbiased, objective estimate of flexibility. The three variables will each range between zero and one, and the model will be the product of all three variables. For the development of the flexibility model, j represents the index of the link, and i the index of the route, one route being the sum of one or more links.

The flexibility model is presented in Eq. (5).

$$F_{(\mathrm{O-D})} = \left(1 - \frac{1}{e^{(N-1)/\delta}}\right) \cdot \left(e^{-\mu * \frac{L_{m,C}}{\sqrt[N]{K_{1,NC} * \cdots * K_{N,NC}}}}\right) \cdot \left(\frac{1}{1 + \mathrm{var}(k_{C_i}/k_{C[\min]})}\right) \tag{5}$$

where $F_{(\mathrm{O-D})}$ is the flexibility model for a given O–D pair; N is number of effective routes between a given O–D (the routes, which are an input and known a priori, are generated by the route-choice model); $L_{m,C}$ is total relative length along all routes in a given O–D pair that is common to more than one route (i.e., when a link is used by more than one route under normal traffic conditions; if only one route uses a certain link, that link is not included in L_m); the lengths are calculated according to Eq. (4), so that the more severe the congestion on a link, the longer is its relative length. $k_{i,NC}$ is absolute length of Route i, which is calculated as the sum of the absolute lengths of all l_j used on route i; k_{c_i} is relative length of route i between a given O–D. $k_{C[\min]}$ is relative length of the shortest route between a given O–D. $\delta,\ \mu$ is the positive parameters to be calibrated.

There are three factors that impact the flexibility model, and these are represented in Eq. (5):

1. The N satisfies requirement 1 and determines that the more routes there are, the more flexible the network system. If there is only one route, the system has no flexibility, and the solution (i.e., network flexibility) will be 0. After sensitivity analysis of several generic examples and estimation of the rate of decline it was initially decided to adopt the value of $\delta = 3$. Parameter 3 was chosen so that the influence of the number of routes will be zero when there is only one route and that influence will increase as a normal logarithmic function until there are about 10 routes. The limit of 10 routes was chosen, since the addition of more routes creates minor changes, with differences among them continuously decreasing.
2. The second variable in Eq. (5) satisfies requirement 5 and gives higher flexibility values to networks having a larger number of individual links (freeways or tollways) that do not have lengths in common with other used routes between the same O–D. The higher the percentage of "independent" links in the network, the greater is its flexibility. The premise here is that the greater the number of independent links between pairs of O–D destinations, the more route-choice options that exist, resulting in more system flexibility. The fact that other O–Ds. might use the same link is taken into account, since the more vehicles that use a given link, the more likely it is that this link will be congested, and therefore its perceived length will increase. Nevertheless, a check of whether a link is "independent" or "common" should always be made in regard to a specific O–D pair. The numerator, $L_{m,C}$, depends on the highway situation; in the case of congestion, it increases; whereas the denominator, $\sqrt[N]{k_{1,NC} \cdot \ldots \cdot k_{N,NC}}$, depends only on the network's topology. The numerator, $L_{m,C}$, might be bigger than the denominator, $\sqrt[N]{k_{1,NC} \cdots k_{N,NC}}$, therefore causing the model to yield zero. Changing the commonality factor to ($e^{-\mu * \frac{L_{m,C}}{\sqrt[N]{K_{1,NC} * \cdots * K_{N,NC}}}}$) can ensure that the decrease will be more moderate as the value of the fracture increases. Sensitivity analysis was made to ensures that the function will decrease sufficiently, but not too fast, as shown in Fig. 3. According to Fig. 3 it was decided to adopt the value of $\mu = 0.3$.
3. The third variable represents the variance among the perceived lengths of each route relative to the shortest route. The more similar the different routes are to the shortest route, the higher will be system flexibility, and the resulting model will be nearer to 1. This result satisfies requirement 4. This variable is sensitive to the relative increase in the routes length, but takes also into consideration the perceived length of the shortest route in the O–D pair. The variance is defined as being equal to infinity in case only one route exists.

It should be noted that all three variables provide a result that is between 0 and 1, and that the flexibility model can receive the value of zero as its lowest value but could only be approaching (asymptotic) 1 as its highest value. The more shared links there are in a given network, the less flexible the network. This phenomenon, which is due to the second variable in Eq. (5), is consistent with and similar to the term "commonality factor" mentioned in Cascetta et al. (1996). In their model, the probability of choosing a route that is more "independent"— i.e., one having fewer common links with other routes—was higher. All else being equal, routes with a higher percentage of mutual links are typically less attractive to drivers, and thus a network with many routes and a high percentage of mutual links decreases the overall flexibility of the network.

Since the v/c ratio and the congestion situation vary throughout the day, flexibility is a dynamic measure that changes as traffic changes.

For each O–D pair, flexibility can be calculated according to Eq. (5). The flexibility model for the entire freeway network—i.e., all possible O–D pairs (P)—is the sum of all individual flexibilities for each O–D pair divided by the sum of all possible O–Ds. in the system as presented in Eq. (6).

$$F_{\text{system}} = \frac{\sum_{\text{O-D}=1}^{P} F_{(\text{O-D})}}{P} \tag{6}$$

It should be noted that the system flexibility model, F_{system}, is dimensionless, and therefore one can use it to compare and evaluate different high-speed, high-capacity networks (freeways and toll roads) regardless of their relative size, number of vehicles in them and length.

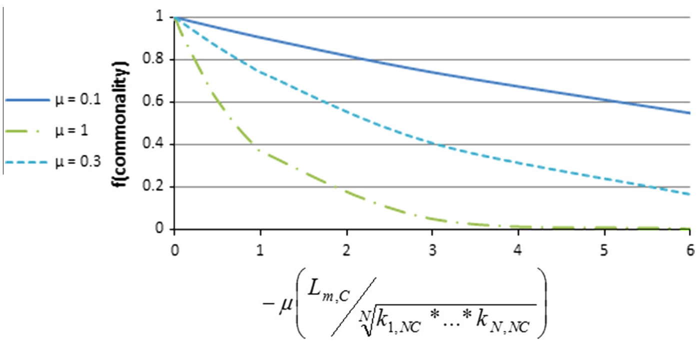

Fig. 3. Change in the second variable in the flexibility model (commonality function) for various a parameters, Eq. (5).

5. Numerical illustrations

In order to illustrate the use of the flexibility model, the model was first applied to schematic "Toy Network" networks without traffic. This was done in order to check for initial reasonability of the results; that is, only the particular network's topology was considered when calculating the flexibility of each of the networks. Four hypothetical networks are presented in Fig. 4; the number next to each link represents the number of that link.

It is clear that Network 1 is the least flexible of the four, since this network has only two routes between O1–D1 and link 3 is common to both these routes. Similar considerations show that the most flexible network (of those shown in Fig. 4) is Network 3, since it has three routes and no mutual links and all links have similar lengths. Network 2 represents a network that is in between Networks 1 and 3, while Network 4 is in between Networks 2 and 3, depending on the length of link 3: the smaller link 3 is in comparison to the route's length, the more similar Network 4 will be to Network 3. On the other hand, the longer link 3 is in comparison to route length, the more similar Network 4 will be to Network 2. When the unit lengths of all links are listed (see Table 1) and the flexibility of each system is calculated by applying Eq. (5) to each of four networks, the resulting flexibilities are as follows: 0.237, 0.283, 0.487, and 0.406, respectively, for Networks 1, 2, 3, and 4.

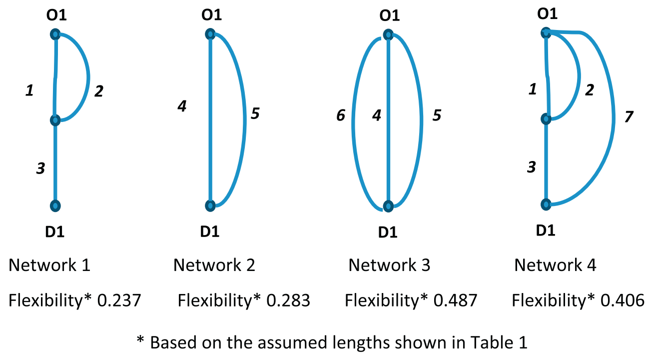

Fig. 4. Comparison of flexibilities of the "Toy Networks".

Table 1
Assumed lengths of links of the "Toy Networks" in Fig. 4.

Link number	Length
1	2
2	2
3	3
4	5
5	5
6	5
7	5

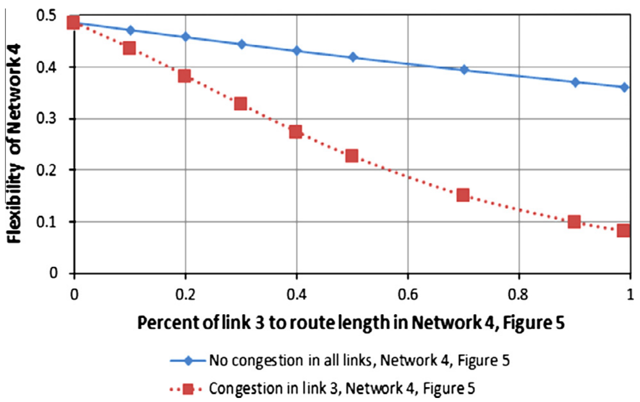

Fig. 5. Flexibility of network 4 in this figure vs. percentage of absolute lengths of link 3 of the total route length.

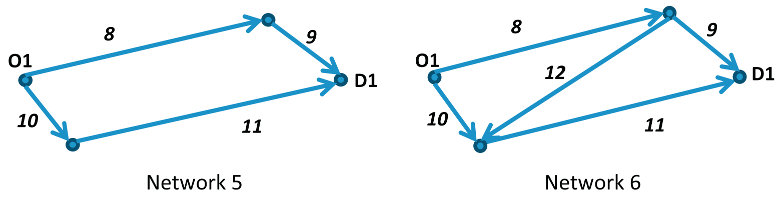

Fig. 6. Networks representing situations similar to the braess paradox.

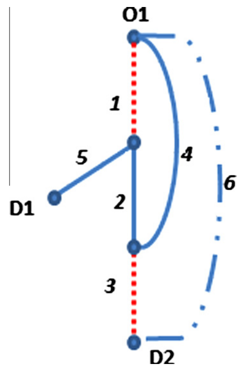

Fig. 7. Example of a schematic network.

Fig. 5 presents the flexibility of Network 4 in Fig. 4 vs. the percentage of the relative length of link 3 of one routes' length in the network.

All three routes are identical in total absolute length. The shorter link 3 is, the more similar Network 4 will be to a network that has only non-common routes, and thus flexibility increases. Congestion on the common link (link 3) means that a network is much less flexible and will remain in that state as long as the percentage of link 3's length increases. This situation is similar to a network with two routes that are almost independent, one route not congested and the second route congested; in other words, there is a big difference in the perceived lengths of the routes, and therefore the third factor of Eq. (5) declines.

Networks 5 and 6 in Fig. 6 represent a situation similar to the Braess paradox, which posits that adding a new road segment to a congested network may actually increase network-wide congestion (Braess, 1968).

In Braess's example, two road segments with low capacity were connected. In our model, the paradox will appear by connecting two segments with long lengths: say, links 9 and 10, which are much shorter than links 8 and 11 in Network 5 (Fig. 6). If link 12 is added by connecting the end of link 8 to the beginning of link 11, the result will be a third route between O1 and D1. To illustrate the paradox, the lengths of links 9 and 10 will be 1 unit, and the lengths of links 8, 11, and 12 will be 50 units. The flexibility of Network 5 with low volume and no congestion is equal to 0.283, while the flexibility of Network 6 with low volume and no congestion is 0.143.

A second hypothetical freeway network is presented in Fig. 7 to illustrate Eq. (6).

The six links in this figure are numbered. The only common link in the network is link 3, which is used by two potential O1–D2 routes when using links 1, 2, and 3 and when using links 4 and 3. When checked for each O–D pair separately, all other links were found to be independent and to serve only one route. Link 1 is not a common link, since it is used once by route O1–D1 and once by O1–D2. Two scenarios with traffic volumes were compared with and without link 6. If the network does not include link 6, there are only three effective routes. The links serving the different O–D routes are presented in Table 2. Clearly, the flexibility of O1–D1 is zero, since there is only one route connecting them.

Table 2
Links belonging to the three routes of the network in Fig. 7.

Link	Link included/not included in		
	O1–D1 (route 1)	O1–D2 (route 2)	O1–D2 (route 3)
1	Yes	Yes	No
2	No	Yes	No
3	No	Yes	Yes
4	No	No	Yes
5	Yes	No	No

Table 3
Traffic and length assumptions for the links in Fig. 7.

Link	Congested?	Original length	Volume	Capacity	Perceived length
1	Yes	6	4000	4800	16.85
2	No	7	1000	2300	7
3	No	8	2000	4800	8
4	No	14	1000	4800	14
5	No	4	3000	4800	4

Flexibility is dependent on the state of congestion on the links and, if they are congested, on the current volume on the links. For illustration, assume that the state of congestion and the volumes at a given time are those listed in Table 3. The information in the columns titled "Original length" and "Capacity" is known a priori and is constant in any given network. The "state" of each link is determined by its COP value; that is, by whether its occupancy and speed are in the convex or concave portion of the speed-occupancy relationship for that link. The link is congested if and only if occupancy on the link is above its COP value according to Eq. (2).

The perceived length of each link is computed according to Eq. (4). The volume of each link is known according to the same observations gathered for Eq. (2). Once the information for Table 2 is gathered, the flexibility of O1–D2 can be calculated according to Eq. (5); in this example, it is equal to 0.253. Therefore, the flexibility of the entire network according to Eq. (6) is 0.127. Once link 6 is added to the network in Fig. 7, with a hypothetical length of 20 units and no congestion, there are three potential routes between O1 and D2. Therefore, the flexibility of O1-D2 increases to 0.433, and system flexibility increases, as well, to 0.216.

6. Application of the flexibility model

As was discussed in the literature review, several previous studies investigated network congestion. The flexibility model can help to quantify perceived network congestion when the flexibility of a network with traffic is divided by an "empty" network (i.e., a network with low volumes that do not fall into the realm of congestion as defined in this study by the COP values of occupancy). The value of that ratio will be between 0 and 1; the closer the ratio is to 1, the more congested

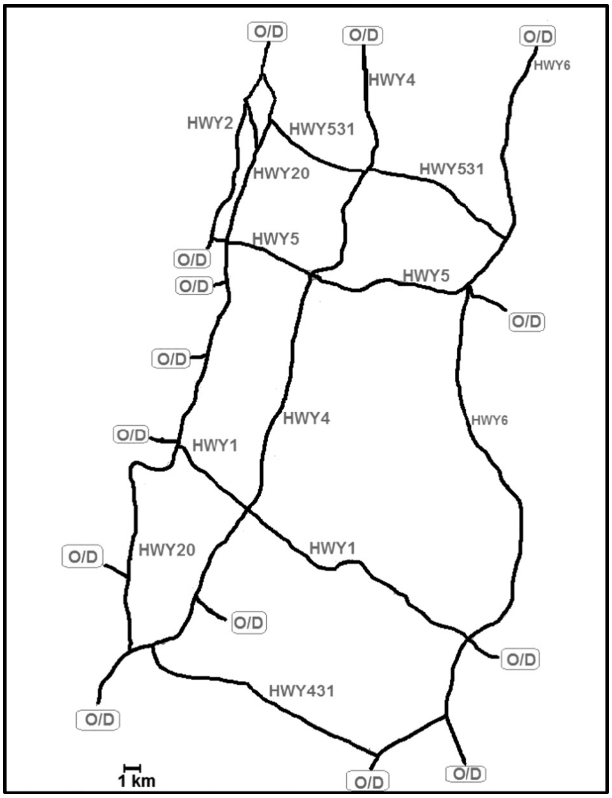

Fig. 8. Schematic map of 8 major freeways in Israel's central region.

the system. Since the flexibility model takes into consideration the topology of a network and the number of different routes composing it, this ratio enables a comparison of different networks in order to objectively quantify the networks that are more congested. The equation for Perceived System Congestion (PSC) is presented in Eq. (7).

$$\text{PSC} = \text{Max}\left\{1 - \frac{F_{C(\text{System})}}{F_{NC(\text{system})}}, 0\right\} \tag{7}$$

where $F_{C(\text{system})}$ is calculated according to Eq. (6), and $F_{NC(\text{system})}$ is the flexibility of a network without traffic or with low volumes in all links.

For example, the network in Fig. 7 without link 6, when examined without vehicles or with low volumes, has a flexibility of 0.127. In cases in which the volume in each link is equal to the volume in Table 3, the flexibility is 0.115. Hence, the measure that assesses the perceived system congestion of this network is estimated as 0.09.

A network consisting of eight major freeways in Israel's central region was tested next. The network includes 57 nodes, representing major intersections, 63 bi-directional freeways as arcs, and 14 O–D pairs. A schematic map of the freeways is presented in Fig. 8.

The relative routes between each O–D pair were found according to the k-shortest-path-link elimination as it appears in the study by Bekhor et al. (2006). The flexibility of the network in Fig. 8 with low volume and no congestion is equal to 0.327. When the same network was tested with volumes equal to those obtained by normal traffic assignment at rush hour, the flexibility was equal to 0.27. During the rush hour, there were 45 congested links. As a result, perceived system congestion during the period tested was 0.175. Because perceived system congestion varies between 0 and 1 (note Eq. (7), with 0 representing no perceived congestion and 1 severe continuous perceived congestion), this result means that the said network is not considerably congested during the rush hour and may need the addition of new links to remedy the flexibility whit low volume, since the network represents during low volume times is relatively insufficient as for the options in the network.

There were 25 O–D pairs of the 182 examined in this example whose flexibility during the rush hour happened to be greater than the flexibility during the low-volume scenario. This might happen when the congested links are the individual ones (i.e., the commonality factor does not change). Because of the prevailing lengths of the links during rush hour, the variability among routes declines in respect to their absolute variability.

7. Summary

This paper shows that the proposed Dynamic Flexibility Model provides a new approach that can estimate a new dimension, that of "system flexibility." When flexibility increases, drivers have more options for selecting a route between origin and destination and for bypassing congested or blocked links, thus improving travel-time reliability. The study also shows that the ratio of network flexibility with traffic to that without traffic may be an estimator of perceived network congestion. Thus, it is possible to perform economic or traffic-flow analyses and to compare different freeway networks with the aid of this model. The concept behind the model adds a new dimension for measuring driver flexibility in choosing routes in case some links suffer congestion or breakdown. Numerous sensitivity analyses were conducted for this study; two hypothetical examples are presented here that show that the proposed model is consistent and simple to use.

This paper contributes to the state-of-the art in presenting a new model for assessing, evaluating, and comparing the flexibility of highway networks. The proposed flexibility model is shown to be dependent on and sensitive to the number of routes between an origin and a destination, as well as to the number of common segments between those routes. Moreover, this flexibility model takes into account the prevailing volume and congestion at any given time and, thus, is dynamic in time. The perceived lengths of a system's links and the amount of commonality among alternative routes are key components of the flexibility model. Employing this model, one can find the amount of flexibility and perceived system congestion in a given network – a concept that enables to calculate the state of perceived congestion in different networks and to compare several freeway networks for their perceived congestion and flexibility, thus enabling the development of measures to increase flexibility and reduce congestion. These aspects have not been addressed in previous studies.

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