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Characteristic length of damage localization in concrete

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ABSTRACT

Computer modeling of crack propagation in concrete requires the knowledge of the characteristic length of failure localization, which coincides with the thickness of the fracture process zone in tension. We propose a simple formula for the calculation of the characteristic length of failure localization. Remarkably, the formula does not require the knowledge of the internal structure of concrete and its components can be derived from the macroscopic experiments only. A trial calculation gives a magnitude of the characteristic length that is in a good agreement with the reported experimental data.

1. Introduction

Fracture is not a peaceful subject, especially in the case of concrete (Shah et al., 1995; Van Mier, 1997; Bazant and Planas, 1998). The material structure of concrete is very sophisticated because its average grain size is large – it is visible. The latter fact makes the essential difference between fracture processes in concrete and, say, nanostructures like graphene, carbon nanotubes, etc. Indeed, nanostructures fail when two adjacent atomic layers separate – Fig. 1 (left). In the case of concrete, however, a crack does not appear as a result of an ideal separation of two adjacent atomic layers. Just the opposite, the crack appears as a result of the development of multiple micro-cracks triggered by the massive breakage of atomic bonds – Fig. 1 (right). The microcracking and the bond breakage are not confined to two neighbor atomic planes: the process involves thousands atomic planes within the representative characteristic volume of size h.

The knowledge of the characteristic size h, where failure localizes, is crucial for numerical simulations of damage necessitated by engineering applications (Hofstetter and Meschke, 2011; Trapper and Volokh, 2010). Indeed, the traditional failure simulations based on the approach of the classical local continuum mechanics are sensitive to the size of geometrical meshes used for the spatial discretization. A way to suppress this pathological mesh-sensitivity is to enforce the characteristic length of the failure localization in the spatial discretization of material. For example, the characteristic length of the failure localization can set the mesh size in the case of finite elements with the linear shape functions. The fixed mesh size is, thus, physically motivated and it should be used in material areas where failure is supposed to localize in cracks and propagate. Alternatively, the characteristic length can be used within the nonlocal gradient- and integral-type theories[1].

2. Characteristic length for concrete

It is not simple to find the characteristic length or, better said, the crack thickness. It is usually attempted to observe this length in experiments where the thickness of the process zone in tension is tracked (Chao et al., 1984; Maji and Shah, 1988; Ouyang et al., 1991; Mindes, 1991; Wittmann, 1992; Guo and Kobayashi, 1993; Li and Shah, 1994; Otsuka and Date, 2000; Denarie et al., 2001). The results scatter depending on the sort of concrete, specimen, grain structure, etc.

In the present note we develop a theoretical approach for a direct calculation of the characteristic length for concrete. The main idea behind the calculation is the following (cf. Volokh, 2011, 2012). Let us assume that the characteristic linear size of the representative volume where bonds break during fracture is h. Then, the work dissipated during the fracture process within the volume is $\sim\omega h^3$ where ω is the density of the *volumetric work of fracture*. In the case of brittle fracture all work is consumed by *the elastic deformation of the breaking bonds*.

On the other hand, the energy of the creation of two surfaces from the bulk is $\sim\gamma h^2$ where γ is the density of the *surface work of fracture* introduced by Griffith (1921).

Equating two works, $\omega h^3 = \gamma h^2$, we get the characteristic length of failure localization

$$h = \frac{\gamma}{\omega}. \tag{1}$$

[1] Unfortunately, the physical interpretation of nonlocal formulations is difficult. For example, gradient formulations increase the order of differential equations and require additional boundary conditions. What is the physical meaning of these additional conditions and how should they be determined? There are no clear physical answers to these questions. Similar is the situation with the integral type theories where the physical meaning of the space averaging is not clear as well as the imposition of boundary conditions is not trivial.

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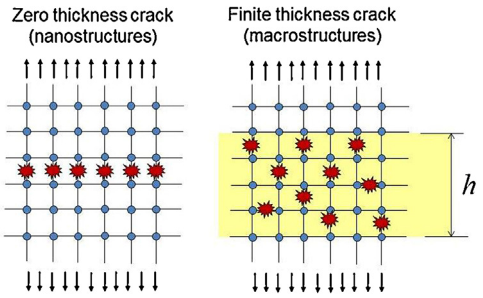

Fig. 1. Cracks of zero and finite thickness.

In the case of concrete we have for the surface work of brittle fracture

$$\gamma = \frac{K_{1C}^2}{E}, \tag{2}$$

where K_{1C} is the mode 1 fracture toughness and E is the Young modulus.

Here we assume that cracks propagate in Mode 1 predominantly.

In order to find the volumetric work of fracture we assume that concrete deforms linearly in tension up to the point of the tensile strength σ_t. At this point the material fails abruptly. In the uniaxial tension the only nonzero component of stress tensor is σ_{11} and the general expression of the volumetric energy density $W=[(1+\nu)\sigma_{ij}\sigma_{ij}-\nu(\sigma_{kk})^2]/2E$ reduces to $W=\sigma_{11}^2/2E$. Thus, the area under the stress–strain diagram up to the tensile strength equals the volumetric work of fracture

$$\omega = \frac{\sigma_t^2}{2E}. \tag{3}$$

Substituting (2) and (3) in (1) we get the general formula for the characteristic length of the failure localization in concrete

$$h = \frac{2K_{1C}^2}{\sigma_t^2}. \tag{4}$$

Unfortunately, there is scattering of the experimental data on the fracture toughness and tensile strength of concrete because of various reasons. Nonetheless, to get a taste of the numbers we choose the following specific magnitudes of the fracture toughness and tensile strength

$$K_{1C} = 0.4\,[\mathrm{MPa}\sqrt{\mathrm{m}}]; \quad \sigma_t = 3.5\,[\mathrm{MPa}]. \tag{5}$$

Then, substituting (5) in (4) we get the following characteristic length

$$h = 2.6\,[\mathrm{cm}]. \tag{6}$$

This number is in a reasonable correspondence with the experimental studies (Chao et al., 1984; Maji and Shah, 1988; Ouyang et al., 1991; Mindes, 1991; Wittmann, 1992; Guo and Kobayashi, 1993; Li and Shah, 1994; Otsuka and Date, 2000; Denarie et al., 2001) which predict the characteristic length h in the range from 1 to 3 cm.

Remark. We emphasize that Eqs. (1) and (4) give the characteristic length of damage localization which is interpreted as the crack thickness – Fig. 1. These formulae should not be confused with the similar ones that have, however, a different meaning. For example, Rice (1968) calculates the length of the process zone ahead of the crack tip using a similar formula; or Espinoza and Zavattieri (2003a, b) calculate the crack opening using a similar formula. It is important that in both mentioned cases cracks have zero thickness which corresponds to the so-called cohesive zone models (CZM). While CZM are very effective for modeling crack propagation when its path is known beforehand (e.g. material interface) their general use for the bulk is more problematic since it requires separate criteria for the crack onset, direction, and branching.

3. Comparison with Bazant's work

To avoid possible misinterpretation of the developed formula we pinpoint the difference between the proposed concept of the volumetric work of fracture, ω, and the concept of the fracture energy density, γ_F, considered by Bazant and Planas (1998). The latter concept is defined as follows (formula (8.2.3) in Bazant and Planas, 1998)

$$\gamma_F = \int_0^\infty \sigma(\varepsilon^f)d\varepsilon^f, \tag{7}$$

where ε^f is "the *inelastic fracturing* strain" that is obtained by the exclusion of the elastic strain σ/E from the total strain.

Thus, Bazant relates volumetric fracture energy to inelastic postpeak deformations on the stress-strain curve with softening (Fig. 8.2.2c in Bazant and Planas, 1998). On the contrary, in the present note inelastic postpeak deformations are ignored and it is assumed that the elastic energy dominates the behavior of the breaking bonds in concrete. It is evident, therefore, that

$$\gamma_F \neq \omega. \tag{8}$$

It is noteworthy that the post-peak deformations used by Bazant can be observed experimentally in the uniaxial tension under the displacement control. Under the force control, which is more relevant to practical situations, the post-peak deformations are not observed because they are statically unstable and, consequently,

$$\varepsilon^f = 0, \quad \gamma_F = 0. \tag{9}$$

4. Discussion

We derived a general formula for the calculation of the characteristic length of the tensile failure localization in concrete – (4). It is amazing that the experiments required for the calculation are macroscopic while the characteristic length is an internal structural parameter of the material.

It is important to note finally that despite its straightforwardness formula (4) was not derived in the previous literature to the best of the author knowledge. It did not happen because the traditional approach of fracture mechanics considered only the surface energy of fracture introduced by Griffith as a material parameter. Traditionally, the volumetric work of fracture was not considered as a material parameter yet it was used by Hillerborg et al. (1976), Bazant and Planas (1998) and others as a variable that should be fitted to the size of the finite element discretization to suppress the pathological mesh-sensitivity of computations. Unfortunately, the Hillerborg approach implies the mesh-dependence of the constitutive equations which is physically meaningless. In the present work, on the contrary, it was assumed that the volumetric work of fracture was a material parameter, similar to the surface work of fracture, implying that the characteristic length of the failure localization was a material parameter as well.

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